Nonlinear Control Exam
February 10, 2016

Student
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1. Let us consider the autonomous Lur'e system in Figure 1

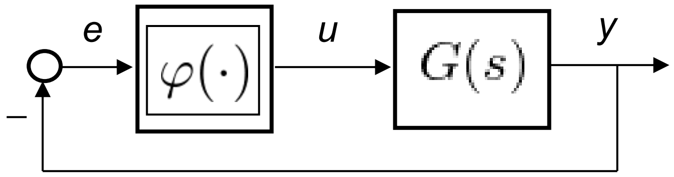

Figure 1: autonomous Lur'e system

where $\varphi(\cdot)$ is a sector nonlinearity and G(s) is the transfer function of a reachable and observable linear system.

Provide clear and precise answers to the following requests:

1.1 Define the notion of absolute stability of the autonomous Lur'e system in sector [0,k], where k>0

1.2 Write the statements of Popov criterion and the circle criterion for the absolute stability of the autonomous Lur'e system in sector [0,k], where k>0, with a graphical interpretation of both criteria. Is there any relation between these criteria?

2. Consider the Lur'e system in Figure 2

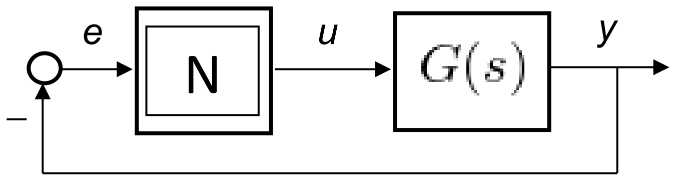

Figure 2: Lur'e system

where

$$G(s) = \frac{10}{(1+10s)^2(1+0.1s)}$$

is the transfer function of a reachable and observable system, and block N is the MB/2 relay with hysteresis in Figure 3

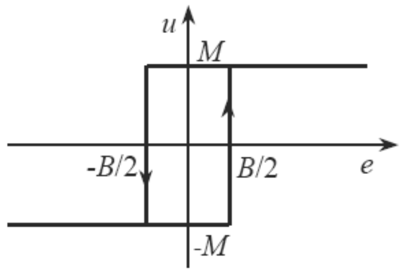

Figura 3: MB/2 relay

Set B/2 =1, and determine the values for M>0 such that the describing function method predicts a permanent oscillation. Evaluate the stability properties of such an oscillation. To this purpose recall that the describing function of the MB/2 relay in Figure 3 is given by

$$D(E) = \frac{2M}{\pi E^2}(\sqrt{4E^2 - B^2} - jB), \; E \geq B/2$$

3. Consider the Lur'e system in Figure 4

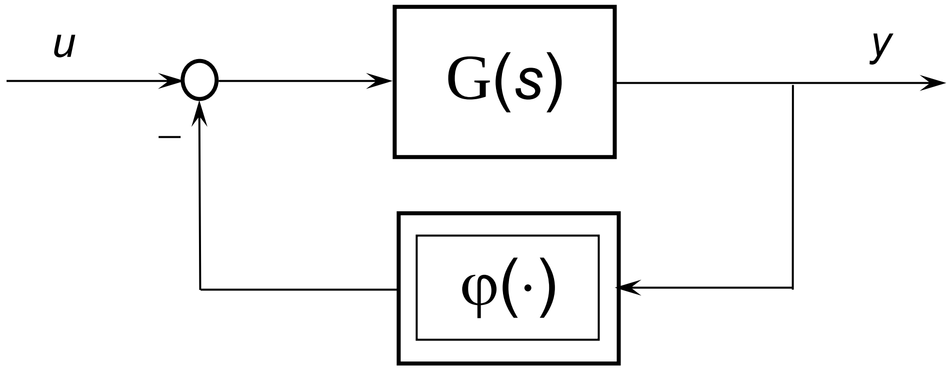

Figure 4: Lur'e system

where

i) $\varphi(\cdot)$ satisfies $0 \leq \varphi(y) \leq 3y, \forall y \in R$

ii) G(s) is the transfer function of a SISO system of order 2 with gain μ>0

$$G(s) = \frac{\mu(1-s)}{(1+s)(1+0.01s)}$$

1. define the notion of L_2 stability for the causal operator H with input u and output y;
2. determine the values for $\mu>0$ such that the operator H with input u and output y is L_2-stable with finite gain by using
    a. the small gain theorem
    b. the circle criterion
3. provide an estimate of the gain of H by the small gain theorem.

4. Given the dynamical system

$$S: \quad \begin{cases} \dot{x}(t) = f(x(t), u(t)), \quad x(0) = x_0 \in \Re^n \\ y(t) = g(x(t), u(t)) \end{cases}$$

$$f(0,0) = 0, \quad g(0,0) = 0$$

Define the notion of passivity and strict passivity.
Let A be a square matrix of size n and B a column vector with n elements. Suppose that there exists a symmetric positive definite matrix P that satisfies

$$A^T P + PA = -I$$

Analyze the passivity and strict passivity properties of the linear dynamical system with transfer function.

5. Describe in a clear and concise way the issue of high frequency oscillations of the control input in a variable structure controller and suggest a possible solution to such an issue.

6. Consider the regular nonlinear SISO system S

$$S: \begin{cases} \dot{x} = a(x) + b(x)u \\ y = c(x) \end{cases}$$

1. Define the notion of relative degree of S in a state x° and the role that it plays in the (local) state feedback linearization of S around x°.
2. Explain what are the conditions to obtain a full state feedback linearization.