Nonlinear Control Exam
March 3, 2016

Student
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1. Let us consider the autonomous time-varying Lur'e system in Figure 1

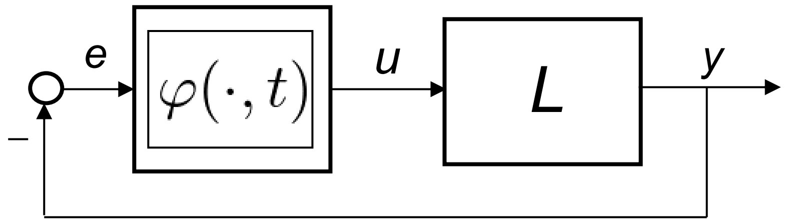

Figura 1: time-varying autonomous Lur'e system

where $\varphi(\cdot,t)$ is a time-varying sector nonlinearity in $[k_1, k_2]$, and L is a reachable and observable linear system

$$L: \begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$$

With reference to such a system

1.1 Define the notion of absolute stability in sector $[k_1, k_2]$
1.2 State the necessary, sufficient, and necessary and sufficient conditions for the absolute stability in sector $[k_1, k_2]$
1.3 Discuss differences and similarities with the case when the sector nonlinearity is time-invariant
1.4 Determine if the system is absolutely stable in sector $[k_1, k_2] = [-1/20, 1/30]$, in the case when L has transfer function

$$G(s) = \frac{10\,(1+s)}{(1+0.2s)^3(1+50s)}$$

2. Consider the autonomous Lur'e system in Figure 2

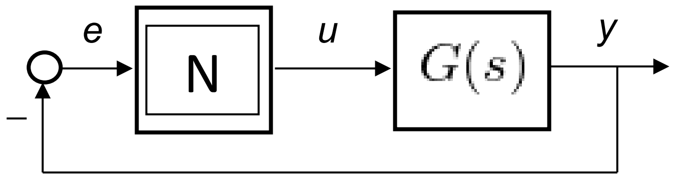

Figure 2: Lur'e system

where G(s) is the transfer function of a reachable and observable system, and block N is the MB/2 relay with hysteresis in Figure 3

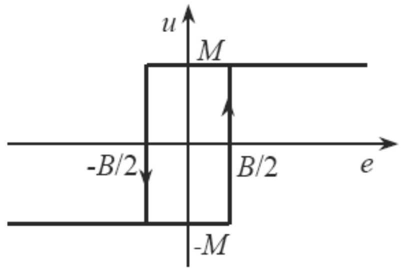

Figura 3: MB/2 relay

with describing function

$$D(E) = \frac{2M}{\pi E^2}(\sqrt{4E^2 - B^2} - jB), \; E \geq B/2$$

With reference to such a system, discuss in some detail the describing function method for the identification and characterization of possible periodic solutions to the system.

3. Consider the Lur'e system in Figure 4

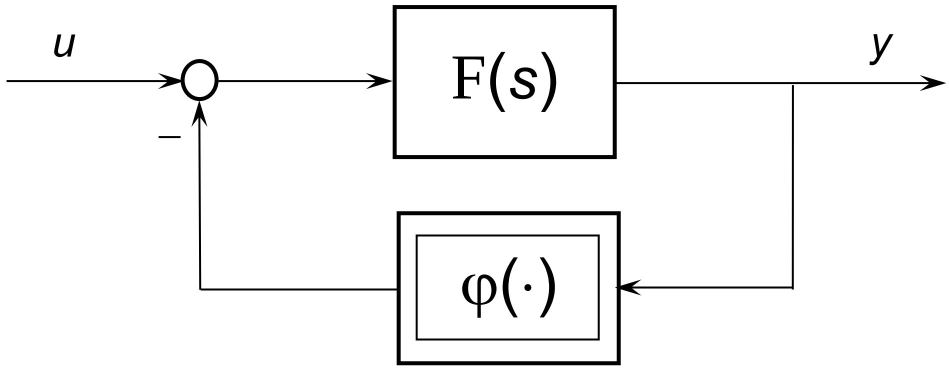

Figure 4: Lur'e system

where

i) $\varphi(\cdot)$ satisfies $0 \leq \varphi(y) \leq ky,\ \forall y \in R$

ii) F(s) is the transfer function of a SISO system of order 2 and is given by

$$F(s) = \frac{3s + 21}{(s+1)(s+10)}$$

3.1 define the notion of L_∞-stability for the operator H with input u and output y;

3.2 by using the small gain theorem, determine the values of $k>0$ such that the operator H with input u and output y is L_∞-stable with finite gain. Provide also an estimate for the gain.

4. Given the dynamical system

$$S: \quad \begin{cases} \dot{x}(t) = f(x(t), u(t)), \quad x(0) = x_0 \in \Re^n \\ y(t) = g(x(t), u(t)) \end{cases}$$

$$f(0,0) = 0, \quad g(0,0) = 0$$

4.1 Define the notion of passivity and strict passivity.

4.2 State the main results that relate the passivity of the system to the stability of the state x=0 when the system has zero input

5. Describe in a clear and concise the variable structure controller design procedure for a linear system and the main characteristics of the resulting system.

6. Consider the regular nonlinear SISO system S

$$S: \begin{cases} \dfrac{dx_1}{dt} = sen(x_2) \\ \dfrac{dx_2}{dt} = -x_1^2 + u \\ y = x_1 \end{cases}$$

6.1 Define the notion of full state feedback linearizability in a state x°

6.2 Verify that system S is full state feedback linearizable in x° = [0 0]'. Design the control law $u = k(x_1, x_2, v)$ via state feedback linearization and pole assignment such that the transfer function from the input v to the output y has all poles equal to -1.