Nonlinear Control Exam
September 30, 2016

Student
Name: ……..
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1. Let us consider the control system in Figure 1

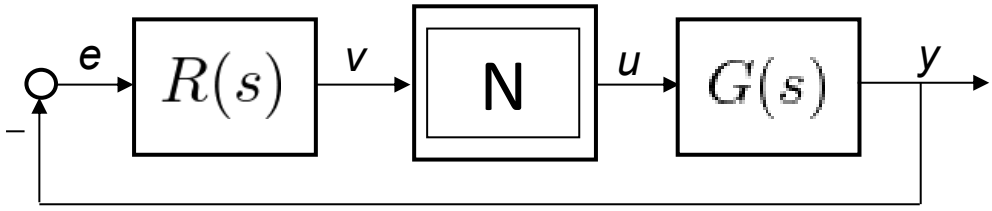
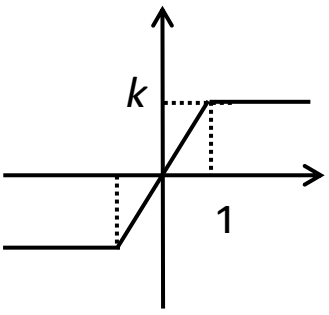
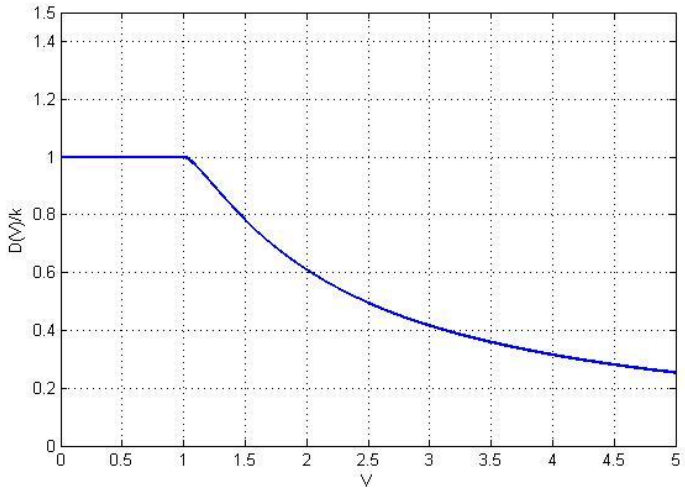


Figure 1: control system

where the nonlinear block is the saturation function plotted below, next to its sinusoidal input describing function





and the transfer function of the system and controller are given by

$$R(s) = \frac{1}{1+sT}$$

$$G(s) = \frac{20}{(1+0.1s)^3}$$

1.1 Describe in a precise yet concise way the describing function method for predicting periodic oscillations.

1.2 Determine if there a value for the parameters *T* and k such that the describing function method predicts a <u>stable</u> limit cycle with v(t) = 2 cos(10t).

2. Let us consider the variable structure control strategy for the output regulation of a linear dynamical system. Explain clearly the issue of the high frequency input switching and a possible solution to such an issue.

3. Given the nonlinear regular SISO system governed by the

$$S_o: \quad \dot{x} = a(x) + b(x)u$$

with a(0) = 0, b(0) ≠ 0, and fully measurable state:

3.1 Provide a necessary and sufficient condition for the system to be fully linearizable in x°=0 via static state feedback.

3.2 Check that such a condition is satisfied for the second order system with

$$a(x) = \begin{bmatrix} x_2^3 - x_1 \\ -x_1 \end{bmatrix} \qquad b(x) = \begin{bmatrix} x_1^2 + 1 \\ 0 \end{bmatrix}$$

(suggestion: try with $y=x_1$ and $y=x_2$) and derive the static state feedback control law that linearizes the system and assigns the poles to be {-1, -10}.

4. Let us consider the autonomous Lur'e system in Figure 2

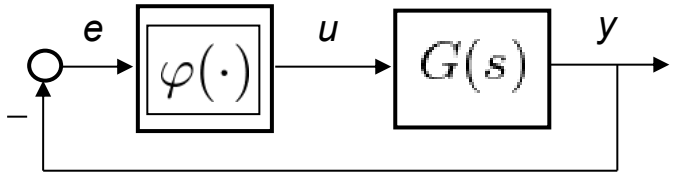


Figure 2: autonomous Lur'e system

where φ(·) is a sector nonlinearity and G(s) is the transfer function of a reachable and observable linear system.

4.1 Define the notion of absolute stability of the autonomous Lur'e system in a sector.

4.2 Write the statement of Popov criterion and the circle criterion for the absolute stability of the autonomous Lur'e system in a sector providing a graphical interpretation of both criteria.

4.3 Describe briefly how the circle criterion can be derived from Popov criterion.

5. With reference to a causal operator $H : \mathcal{L}_e \to \mathcal{L}_e$

5.1 define the notions of boundedness, and weakly boundedness, zero-bias gain and gain, and describe the connection between notions and gains.

5.2 define when an operator is affine and what are the key characteristics of an affine operator.