Nonlinear Control
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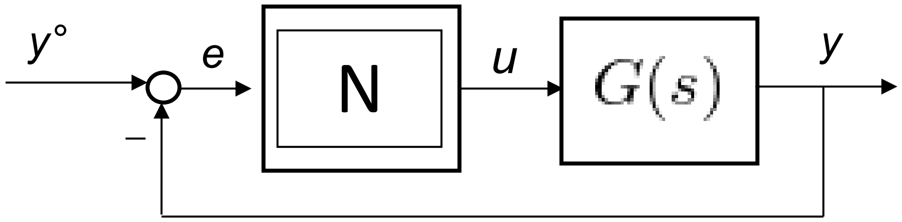
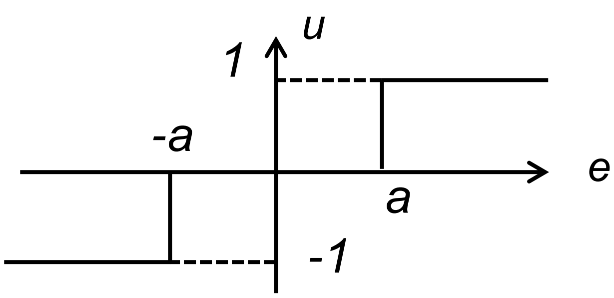
1. Consider the following feedback scheme



where

$$G(s) = \frac{200}{(s+10)^2(s+1)\,(1+0.001s)}$$

is the transfer function of an observable and reachable linear system and the nonlinear component is the relay with deadband reported in the figure below

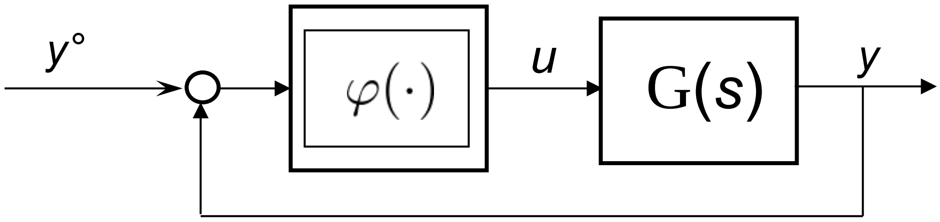


Set y°(t)=0, t ≥ 0, and determine how parameter a should be set so that the describing function method predicts some limit cycle.

To this purpose, please recall that the sinusoidal-input describing function of the considered relay with deadband is

$$D(E) = \begin{cases} 0, & E < a \\ \frac{4}{\pi E}\sqrt{1 - \frac{a^2}{E^2}}, & E \geq a \end{cases}$$

2. Let us consider the nonlinear system in the following figure



where $\varphi(\cdot)$ is a sector nonlinearity in [-2k, k], with k>0, and

$$G(s) = \frac{2\,(1 - 0.01s)}{(1+s)^2(1+0.001s)}$$

is the transfer function of a linear system of order n=3.

2. 1 Define the L_2 stability notion for the operator H with input y° and output y.

2.2 State the small gain theorem and the circle criterion for the L_2 stability of the operator H with input y° and output y.

2.3 Determine the maximum value for k>0 such that the operator H with input y° and output y is L_2 stable via (a) the small gain theorem and (b) the circle criterion for the L_2 stability of H, specifying in a clear a precise way the adopted procedure and motivating the obtained result.

2.4 How the reply to 2.3 would change if the operator with input y° and output u were considered?

3. With reference to variable structure control of a linear dynamical SISO system,

3.1 draw the control scheme and explain the role of the various components;

3.2 describe in a clear and concise way the robustness properties of the scheme with respect to a bounded load disturbance acting additively on the system control input.

4. Consider the nonlinear system

$$S: \quad \begin{cases} \dot{x}(t) = f(x(t), u(t)), \quad x(0) = x_0 \in \Re^n \\ y(t) = g(x(t), u(t)) \end{cases}$$

$$f(0,0) = 0, \quad g(0,0) = 0$$

4.1 Define the notion of passivity

4.2 State the main results regarding the relation between passivity and the stability properties of the equilibrium x=0 associated with zero input

5. Consider a regular nonlinear SISO system S described by

$$S: \begin{cases} \dot{x} = a(x) + b(x)u \\ y = c(x) \end{cases}$$

5.1 Define the notion of state feedback linearization.

5.2 Provide an example of fully state feedback linearisable and partially state feedback linearisable system.