Nonlinear Control
June 30, 2016

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1. Let us consider the autonomous Lur'e system in Figure 1

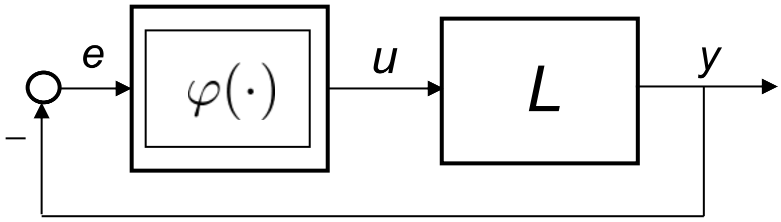

Figure 1: autonomous Lur'e system

where φ(·) is a sector nonlinearity in [0, k], and L is a reachable and observable linear system with transfer function

$$G(s) = \frac{2}{(1+s)^2(1+0.01s)}$$

(a) With reference to such a Lur'e system

1.a.1 state the Popov criterion for the absolute stability in sector [0, k]

1.a.2 provide a geometric interpretation of Popov criterion by drawing (approximately) Popov plot of G(s)

1.a.3 explain what is meant by the Aizerman conjecture and show how it can be verified graphically

(b) Discuss differences and similarities in terms of necessary and sufficient conditions with the case when the sector nonlinearity in [0, k] is time-varying , i.e., φ is a function of time φ(·,t).

2. Consider the Lur'e system in Figure 2

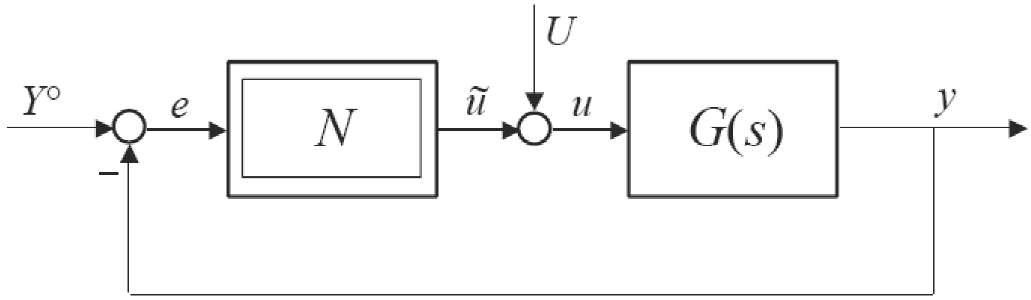

Figure 2: Lur'e system

Suppose that all inputs of the Lur'e system are constant.

Answer in a clear and concise way the following questions:

2.1 define the notions of sinusoidal input and dual input describing functions
2.2 illustrate the describing function method for the identification and characterization of possible periodic solutions to the system.

3. Consider the linear system S described by the following equations

$$\begin{cases} \dot{x}_1 = -2x_2 + u \\ \dot{x}_2 = 2\,x_1 \end{cases}$$

Set $s(x) = x_1 + \beta x_2$, where β is a scalar parameter.

(a) determine the set B of values for β such that system S converges to a uniquely defined (pseudo-)equilibrium $\bar{x}$ = [0 0]', when constrained to evolve on the surface $s(x) = 0$, starting from an arbitrary point on that surface.
(b) determine the value $\bar{\beta}$ for β in the set B such that the time constant of the asymptotically stable dynamics of the reduced order system on the surface s(x)=0 is equal to 1.
(c) Set $\beta = \bar{\beta}$ and design a state-feedback variable structure controller that makes the system reach the sliding surface $s(x) = 0$ within time t=10 when x(0) =[2 2]'.
(d) draw the block diagram of the designed closed-loop control scheme.

Suppose now that only the first state variable is available, i.e., $y = x_1$

(e) verify that the system

$$\begin{cases} \dot{x}_1 = -2x_2 + u \\ \dot{x}_2 = 2\,x_1 \\ y = x_1 \end{cases}$$

is observable

(f) draw the scheme of the modified output-feedback variable structure controller

4. Consider the nonlinear systems in Figure 3

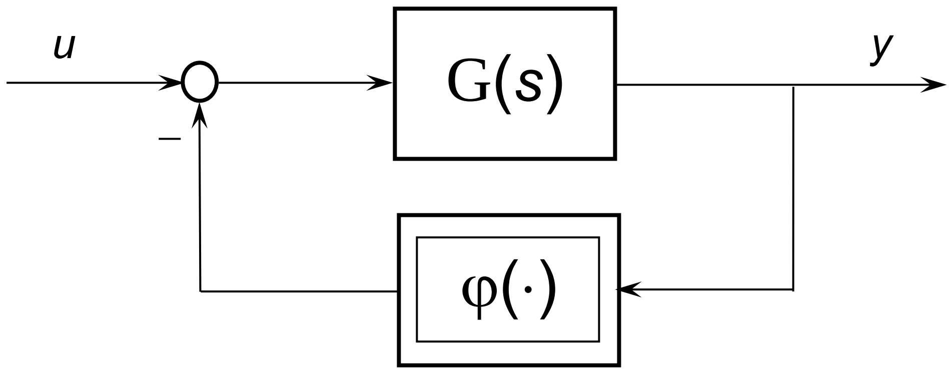

Figure 3: sistema di Lur'e

where $\varphi(\cdot)$ is a sector nonlinearity, and G(s) is the transfer function of an asymptotically stable, strictly proper reachable and observable linear system.

4.1 Define the L_2 stability notion for the causal operator H with input u and output y

4.2 State the small gain theorem and the circle criterion for the L_2 stability of H, specifying the conditions under which they are valid.

5. Consider a regular nonlinear SISO system S described by

$$S: \begin{cases} \dot{x} = a(x) + b(x)u \\ y = c(x) \end{cases}$$

5.1 Define the notion of full state feedback linearization

5.2 Explain when a system is fully state feedback linearisable and what is the procedure to linearize it