Nonlinear Control
February 9, 2017

Student Name: ……………………………………………………

Personal ID number: ……………………………………………………

1. Consider the Lur'e time-varying system in Figure 1

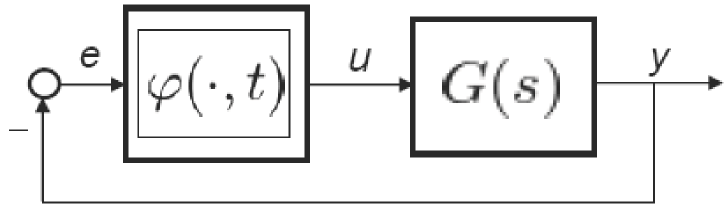


Figura 1: sistema di Lur'e autonomo

where $\varphi(\cdot,t)$ is a time-varying sector nonlinearity in $[k_1, k_2]$, whereas G(s) is the transfer function of a SISO reachable and observable linear system.

1.1. Define the notion of absolute stability and briefly motivate the problem of studying absolute stability for such a structured system.

1.2. State necessary and/or sufficient conditions for the absolute stability of the above system, poiting out the possible differences with the case of a Lur'e system with time-invariant sector nonlinearity.

1.3. Discuss possible connections between absolute stability analysis of a Lur'e time-varying system and switches linear systems stability analysis.

2. Consider a switched system

$$\dot{x} = f_\sigma(x)$$

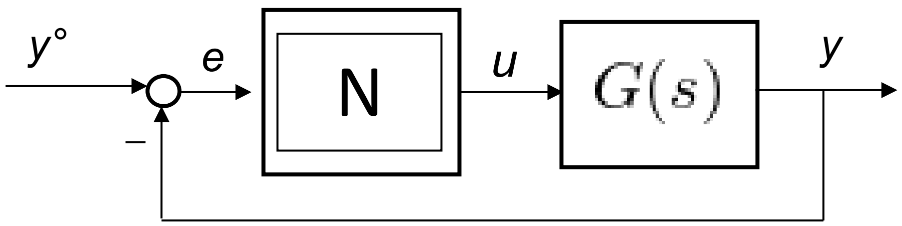
where

$$\dot{x} = f_p(x), \; p \in \mathcal{P} = \{1, 2, \ldots, m\}$$

is a family of systems with $f_p(0)=0$ and σ is the switching signal

Provide conditions for the equilibrium x=0 of the switched system to be globally asymptotically stable, uniformly with respect to the switching signal σ, in the general case when the family of systems is nonlinear and when it is linear.

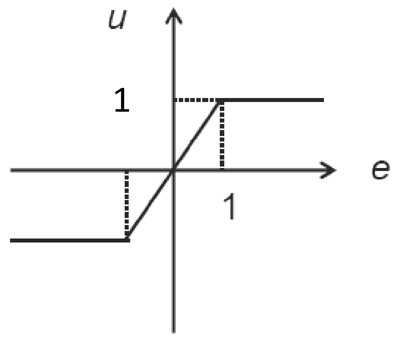
3. Consider the following feedback scheme



where

$$G(s) = \frac{\mu}{(1+1000s)^2(1+Ts)}, \qquad T > 0$$

is the transfer function of an observable and reachable linear system and the nonlinear component is the saturation function reported in the figure below



Set y°(t)=0, t $\geq$ 0.

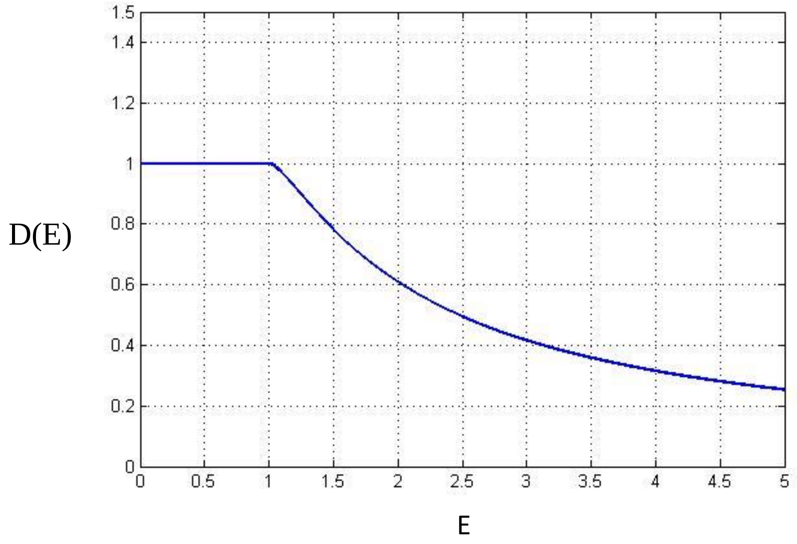
3.1 determine how parameters *k* and *T* should be set so that the describing function method predicts some limit cycle with $e(t) = 3\sin(t)$

3.2 evaluate if the predicted limit cycle is stable.

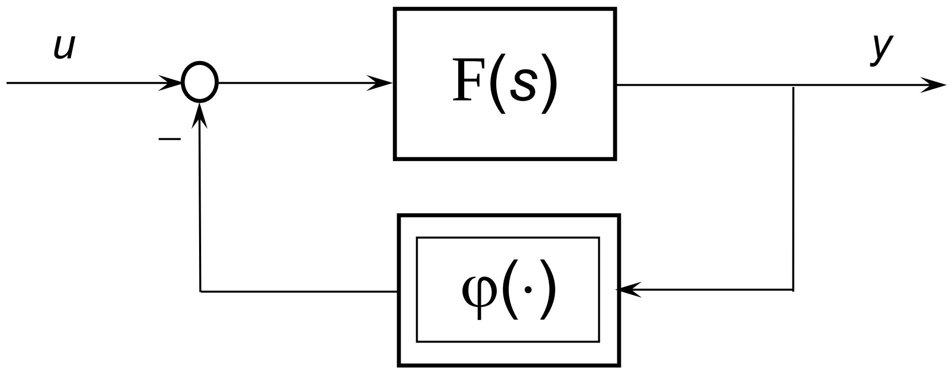
Recall that the sinusoidal-input describing function of the saturation function is

$$D(E) = \begin{cases} 1, & E \leq 1 \\ \frac{2}{\pi}\left(arcsen(\frac{1}{E}) + \frac{1}{E}\sqrt{1-(\frac{1}{E})^2}\right., & E > 1 \end{cases}$$

as reported in the figure below.



4. Consider the following Lur'e system



where

i) φ(·) is s sector nonlinearity in [0,k]

ii) F(s) is the transfer function of a SISO system of order 2 and is given by

$$F(s) = \frac{10(1 + 10s)}{(s + 1)(1 + 0.001s)}$$

4.1 define the notion of L_2-stability for the operator H with input u and output y;

4.2 by using the small gain theorem, determine the values of $k>0$ such that the operator H with input u and output y is L_2-stable with finite gain;

4.3 by using the circle criterion, determine the values of $k>0$ such that the operator H with input u and output y is L_2-stable with finite gain. Compare the obtained value with the previous one.

5. Consider a regular nonlinear SISO system S described by

$$S: \begin{cases} \dot{x} = a(x) + b(x)u \\ y = c(x) \end{cases}$$

5.1 Define the notion of state feedback linearization.

5.2 Provide an example of fully state feedback linearisable and partially state feedback linearisable system.