Nonlinear Control
June 28, 2017

Student Name: ……

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1. Consider the Lur'e system in Figure 1

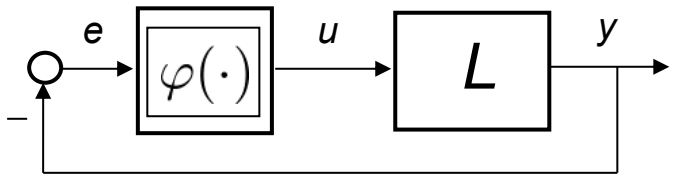


Figure 1: autonomous Lur'e system

where L is a completely observable and reachable linear system with transfer function

$$F(s) = \frac{10}{(1+s)^4}$$

and $\varphi(\cdot)$ is a sector nonlinearity.

(a) Define the notion of absolute stability in the sector $[k_1,k_2]$
(b) Explain clearly Aizerman conjecture.
(c) Determine if Aizerman conjecture is valid for the considered Lur'e system with reference to sector [0,k].

2. Consider the Lur'e system in Figure 2

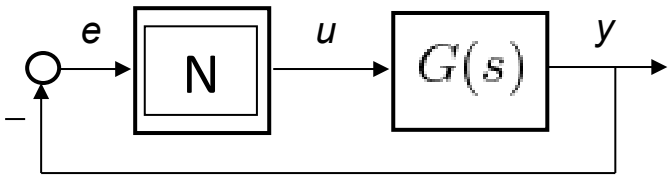


Figure 2: Lur'e system

where $G(s)$ is the transfer function of a completely observable and reachable linear system and N is the MB/2 relay with hysteresis in Figure 3.

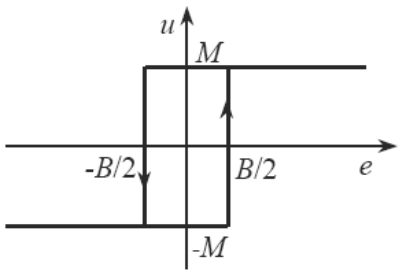


Figure 3: MB/2 relay with hysteresis

Set B/2=1 and explain clearly how one can use the describing function method to determine for what values of M>0 there exists a limit cycle in the closed-loop system of Figure 2.

To this purpose recall that the sinusoidal input describing function of the MB/2 relay with hysteresis is given by:

$$D(E) = \frac{2M}{\pi E^2}(\sqrt{4E^2 - B^2} - jB),\ E \geq B/2$$

3. Consider the linear system S described by

$$\begin{cases} \dot{x}_1 = -10\, x_2 + \mathrm{u} \\ \dot{x}_2 = \ \ 10\, x_1 \end{cases}$$

Set $\mathrm{s}(x) = 2x_1 + x_2$.

(a) show that system S converges to a uniquely defined (pseudo-)equilibrium $\bar{x}$ when constrained to evolve on the surface $\mathrm{s}(x) = 0$, starting from an arbitrary point on that surface. Determine $\bar{x}$.

(b) design a state-feedback variable structure controller that makes the system reach the sliding surface $\mathrm{s}(x) = 0$ within time t=10 when x(0) =[1 1]'.

(c) draw the block diagram of the designed closed-loop control scheme.

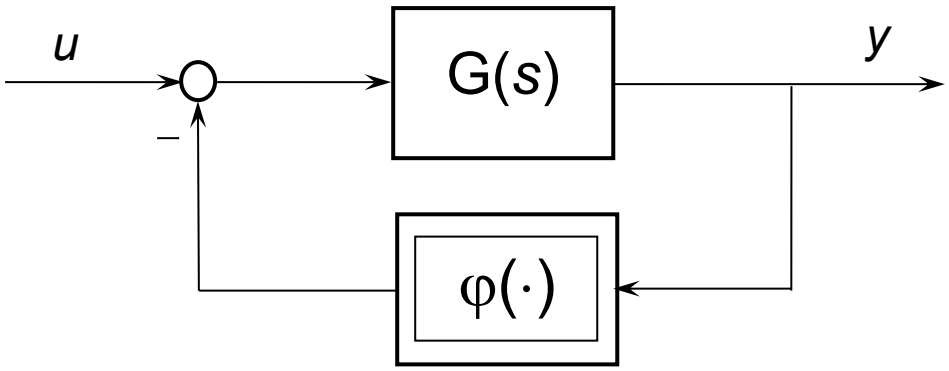
4. Consider a regular nonlinear SISO system S described by

$$S: \begin{cases} \dot{x} = a(x) + b(x)u \\ y = c(x) \end{cases}$$

(a) Define the notion of relative degree of S in a state x° and its role when (locally) linearizing the system by static state feedback.

(b) Provide an example of fully state feedback linearisable and partially state feedback linearisable system.

5. Consider the Lur'e system



where G(s) is the transfer function of an asymptotically stable strictly proper linear system, reachable and observable, while $\varphi(\cdot)$ is a sector nonlinearity.

(a)Define the notion of L_2 stability.

(b) State the circle criterion and the small gain theorem for analyzing the L_2 stability the system above.