Nonlinear Control
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1. Consider the Lur’e system in Figure 1

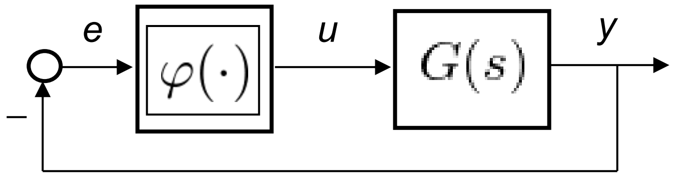

Figure 1: Autonomous Lur’e system

where G(s) is the transfer function of a completely observable and reachable linear system and $\varphi(\cdot)$ is a sector nonlinearity.

With reference to such a system, provide a clear and motivated answer to the following requests:

1.1 Define the notion of absolute stability in the sector $[k_1,k_2]$;
1.2 State the circle criterion for the absolute stability in the sector $[k_1,k_2]$;
1.3 Explain what is the relation between the circle criterion for the absolute stability in the sector $[k_1,k_2]$ and the Popov criterion for the absolute stability in the sector [0,k], k>0.
1.4 Is it possible that a Lur’e system satisfies the Popov criterion for the absolute stability in the sector [0,k], k>0, but not the circle criterion? What about the opposite?

2. Consider the nonlinear dynamical system described by the following equations

$$\begin{cases} \dfrac{dx_1}{dt} = x_1 + x_2 \\ \dfrac{dx_2}{dt} = -x_1^3 - x_1 - x_2 + u \\ y = x_1 + x_2 \end{cases}$$

2.1 Is the system zero-state observable?

2.2 Check if the system is passive with storage function

$$V(x_1, x_2) = \frac{(x_1+x_2)^2}{2} + \frac{x_1^4}{4}$$

2.3 Check that $x_1 = x_2 = 0$ is an equilibrium for the system associated with zero input (u=0) and evaluate its stability properties

2.4 Consider the feedback system with u=-ky+v. Evaluate if $x_1 = x_2 = 0$ is a globally asymptotically stable equilibrium of the zero-input feedback system for any $k > 0$.

3. Discuss in a precise yet concise way the describing function method: framework, objective, methodology.

4. With reference to the output regulation problem for a linear system with a constant reference signal,

4.1 describe the solution based on the sliding mode control approach, detailing the main steps

4.2 draw the sliding mode control scheme

4.3 discuss in a clear and concise way the issue of control input oscillations in the sliding mode control approach and a possible solution

5. Consider the SISO nonlinear regular system of order n, described by

$$S_o : \quad \dot{x} = a(x) + b(x)u$$

5.1 state the necessary and sufficient condition for (local) state feedback full linearization in a given state $x^o \epsilon R^n$

5.2 determine if the system of order n=2 with

$$a(x) = \begin{bmatrix} 2\text{sen}(x_2) + x_1 - 1 \\ -x_1^2 \end{bmatrix} \qquad b(x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

is locally fully linearizable by state feedback in $x^o = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

If that is the case, write the state feedback control law that makes the feedback system linear with eigenvalues all identical to -1.