POLITECNICO DI MILANO

Distributed algorithms for optimization and control over networks

DEIB PhD Course, Politecnico di Milano

February 10-14, 2020

Lecturers

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Politecnico di Milano, Italy

Organizer

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A request from the PhD School

Those of you who registered to the course as “single course attendees” should contact the PhD secretariat to officially appear in the list of attendees.

Goal of the course

Introduce to the analysis and design of distributed decision making schemes for multi-agent systems seeking convergence to an optimal cooperative solution.

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Motivating applications: energy and transportation systems

Structure and content outline

The course is structured in 4 parts

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Part 1: Motivation and illustrative applications

Introduction to decision making problems arising in smart grid control and optimization, and in coordination and control for electric vehicle fleets.

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Part 1: Motivation and illustrative applications

Introduction to decision making problems arising in smart grid control and optimization, and in coordination and control for electric vehicle fleets.

Part 2: Mathematical tools

Introduction to the mathematical tools of graph theory, convex analysis, optimization, duality theory that constitute the theoretical backbone for the analysis and design of cooperative algorithms

Structure and content outline

Part 3: Distributed cooperative algorithms

Primal-based and dual-based algorithms for distributed cooperative decision making will be illustrated, resting on the math tools of Part 2

Part 4: Distributed optimization in uncertain networks

The algorithmic solutions described in Part 3 will be extended to the case when the multi-agent optimization problem is affected by uncertainty.

Exam

Students will be evaluated based on a small project or on the study of an advanced topic related to the course.

Project and topic should be agreed upon with the organizer of the course.

Schedule

Monday, February 10

11:30 – 13:00 Motivation [MP]

14:30 - 17:00 Math tools [MP]

Tuesday, February 11

09:30 – 11:00 & 11:15 – 12:00 Math tools [MP]

14:30 - 16:00 Math tools [KM]

Wednesday, February 12

09:30 – 11:00 & 11:15 – 12:00 Primal-based algo [KM]

14:30 - 16:00 Primal-based algo [KM]

Thursday, February 13

09:30 – 11:00 & 11:15 – 12:00 Duality-based algo [AF]

14:30 - 16:00 Duality-based algo [AF]

Friday, February 14

09:30 – 11:00 & 11:15 – 12:00 Distributed uncertain opt [SG]

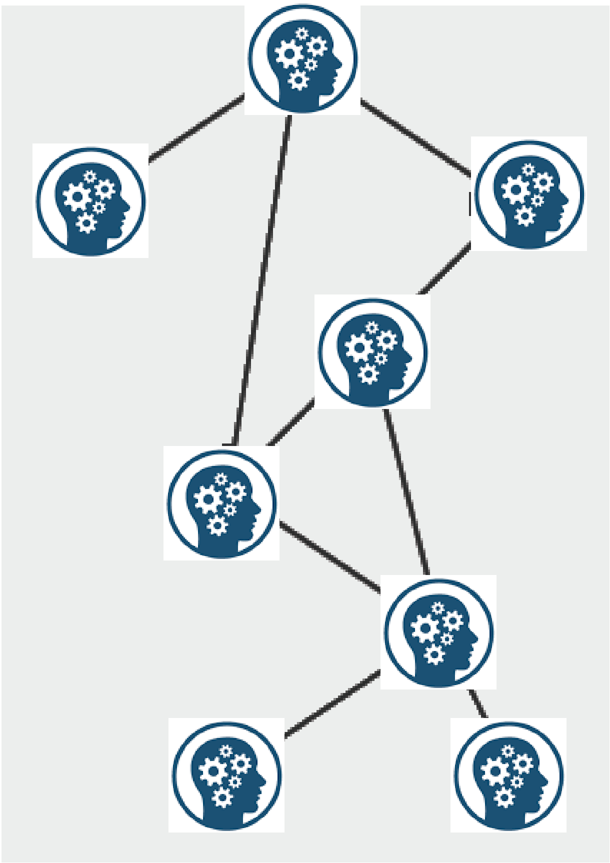
14:30 - 16:00 Distributed uncertain opt [SG]

Motivation and context

Large-scale system composed of multiple sub-systems (agents) with

- computational power
- communication capabilities

Motivation and context



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Agents can communicate according to a given communication graph to possibly agree on a solution to a coupled decision problem.

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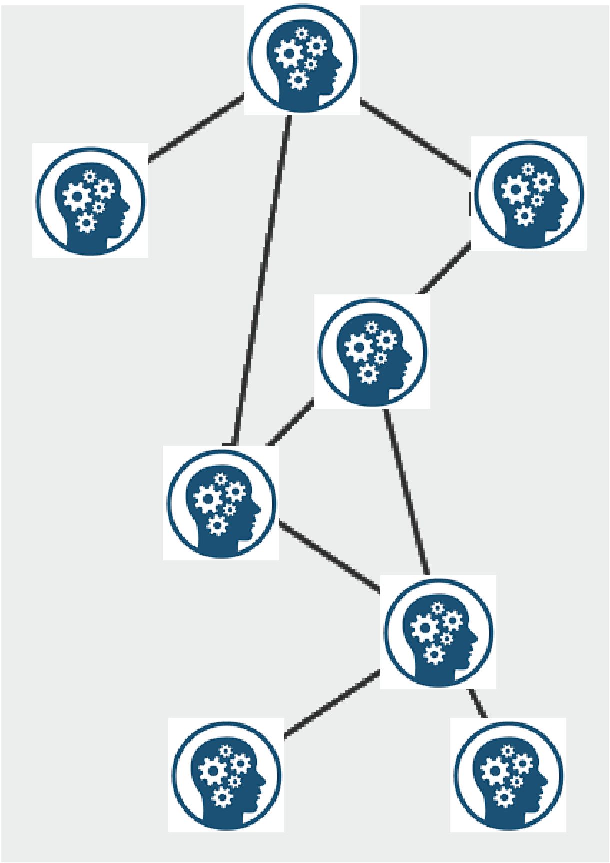

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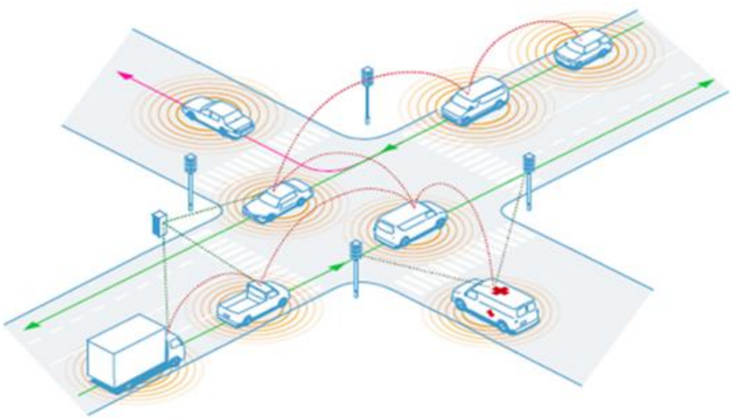
→ **network system**

Motivation and context

Transportation systems



Energy systems

Robotic network

Social network

Networks and decision making challenges

Network

- Large scale systems
- Multi-agent – multiple interacting entities/users
- Heterogeneous – different physical or technological constraints per agent; different objectives per agent

Networks and decision making challenges

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- Computation: problem size too big

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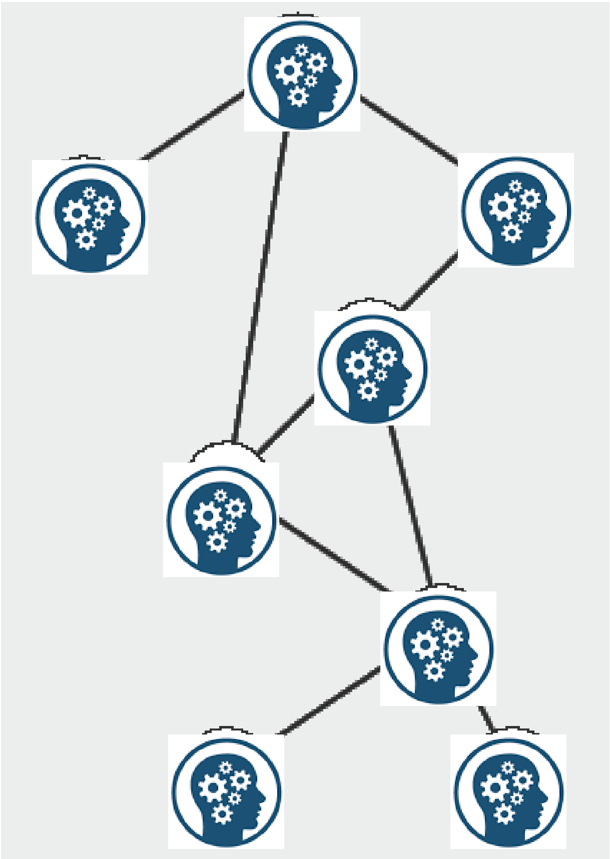
- Heterogeneous – different physical or technological constraints per agent; different objectives per agent

Challenges

- Computation: problem size too big
- Communication: not all communication links at place; link failure; communication graph given, not to be designed
- Information privacy: agents may not want to share information with everyone

Cooperative decision making over networks



Large-scale system composed of multiple **cooperative agents** with

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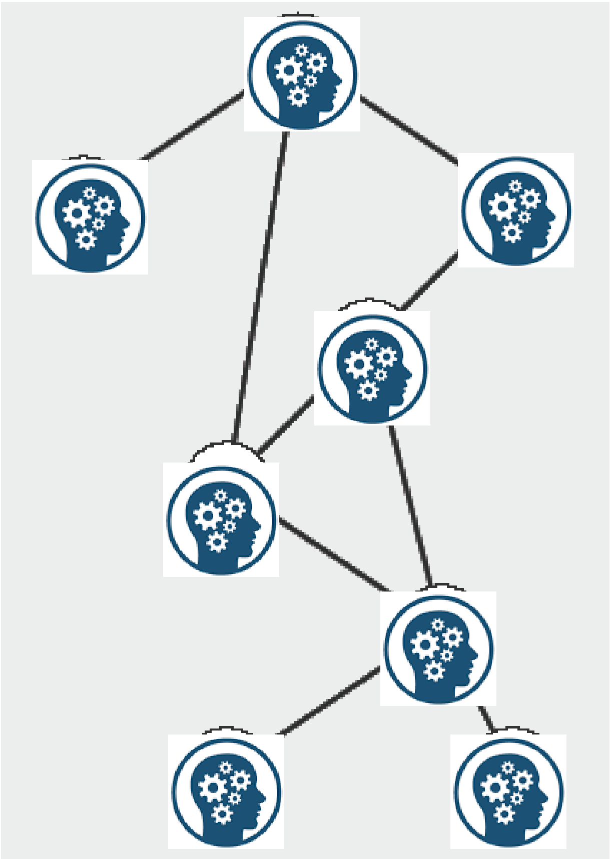
Cooperative decision making over networks



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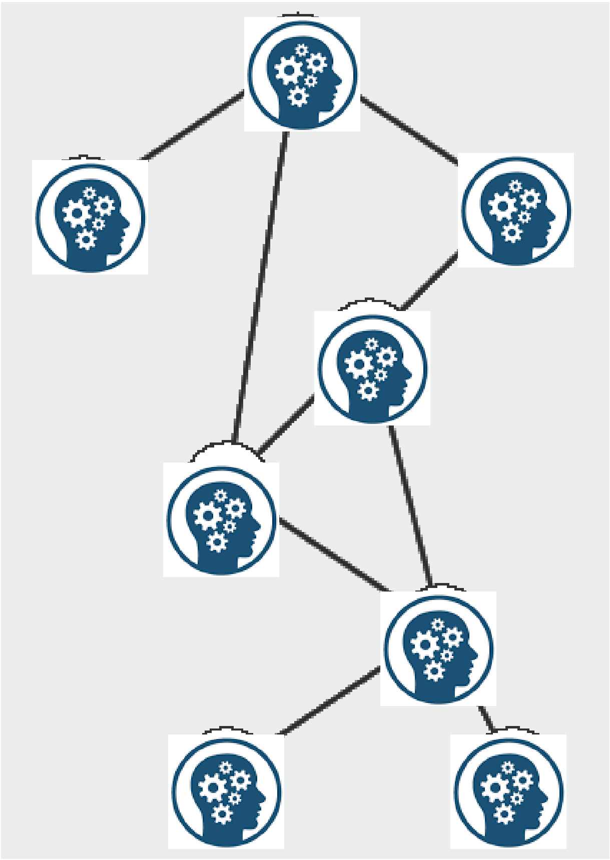
Goal

Solving optimally a coupled decision making problem involving the overall network, so as to get the social welfare solution

Cooperative decision making over networks



The decision making problems can typically be formulated as an

optimization program for the overall system

where a cost function, typically the sum of local costs, is minimized subject to

- coupled decisions
- local information
- communication constraints

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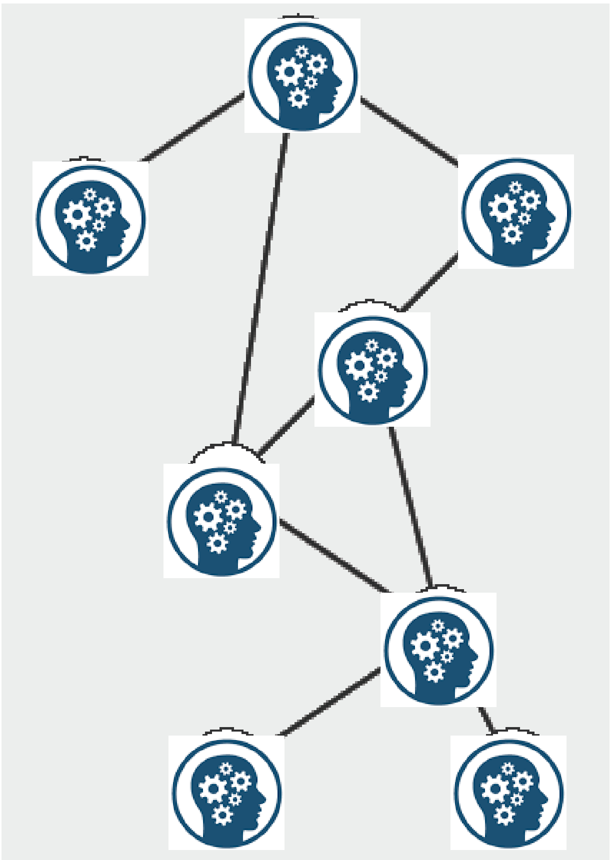

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If the optimization problem has a separable structure, then, a distributed solution can be adopted

What is a distributed solution?

The large scale optimization problem is addressed by distributing computations locally to the agents.

Each agent iteratively solves a smaller optimization problem based on its local information (local objective and constraints) and the information received from its neighbors, till convergence, ideally to the global optimum

Why a distributed solution?

1. Scalable methodology
   - Communication: Only between neighbors
   - Computation: Only local; in parallel for all agents

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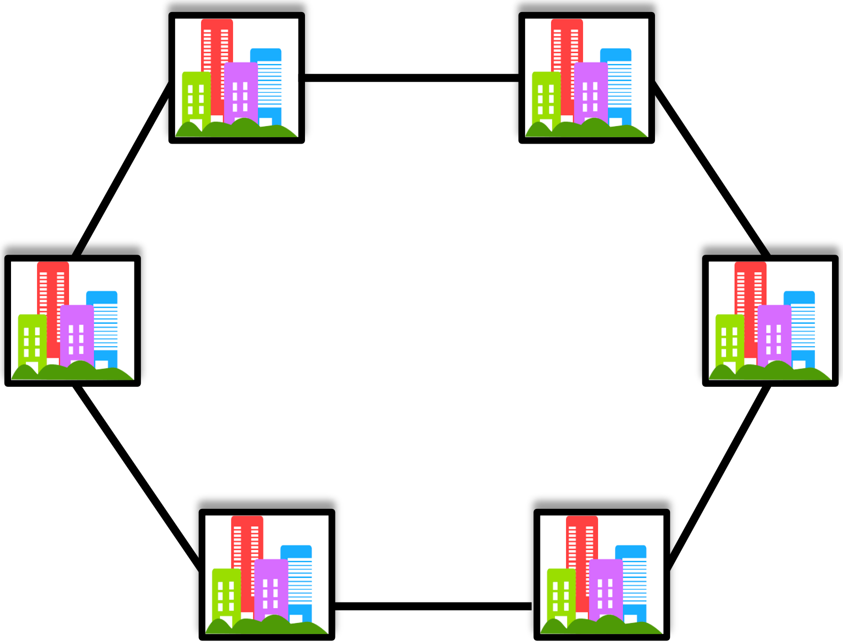
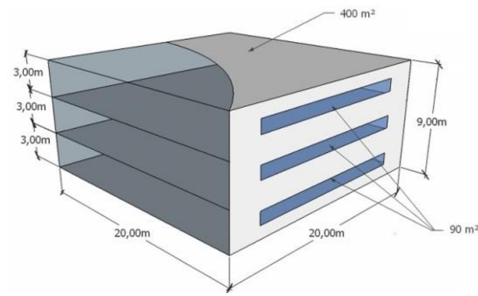
4. Numerous applications
   - Wireless networks
   - Optimal power flow
   - Electric vehicle charging control
   - Energy management in building networks

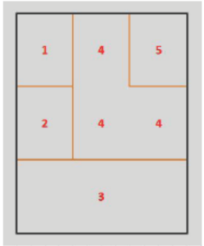
Energy management in a building network



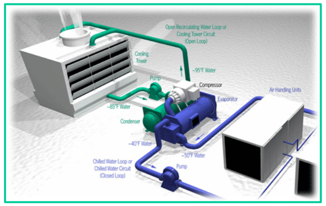
Building network

- m buildings, divided into zones





- a chiller unit for each building



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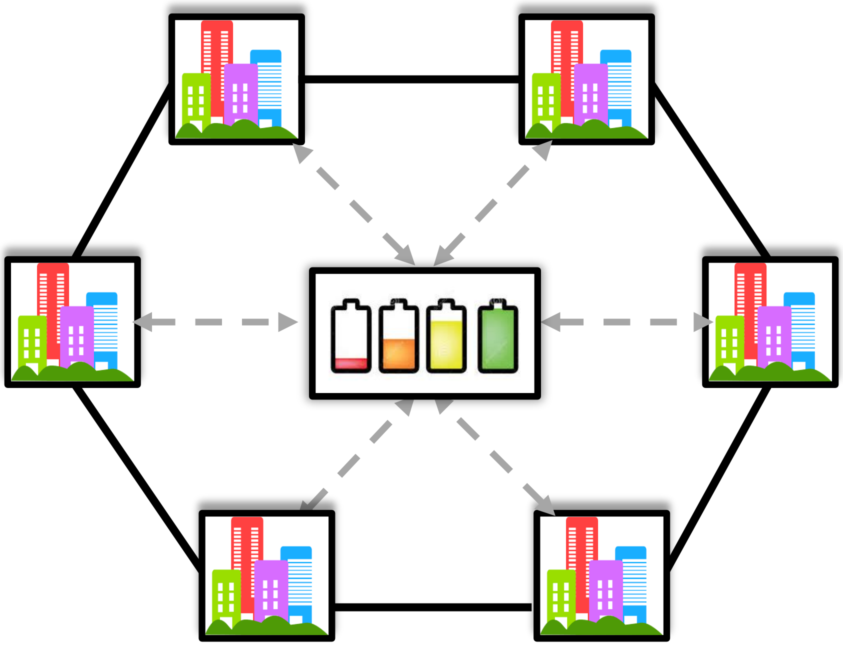

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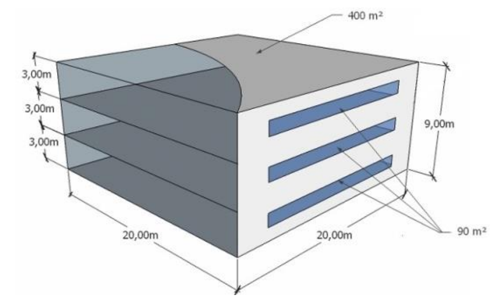

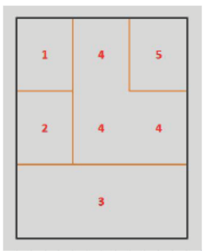

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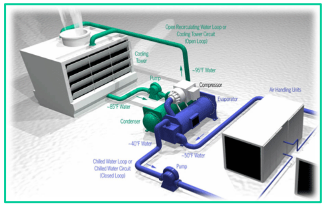
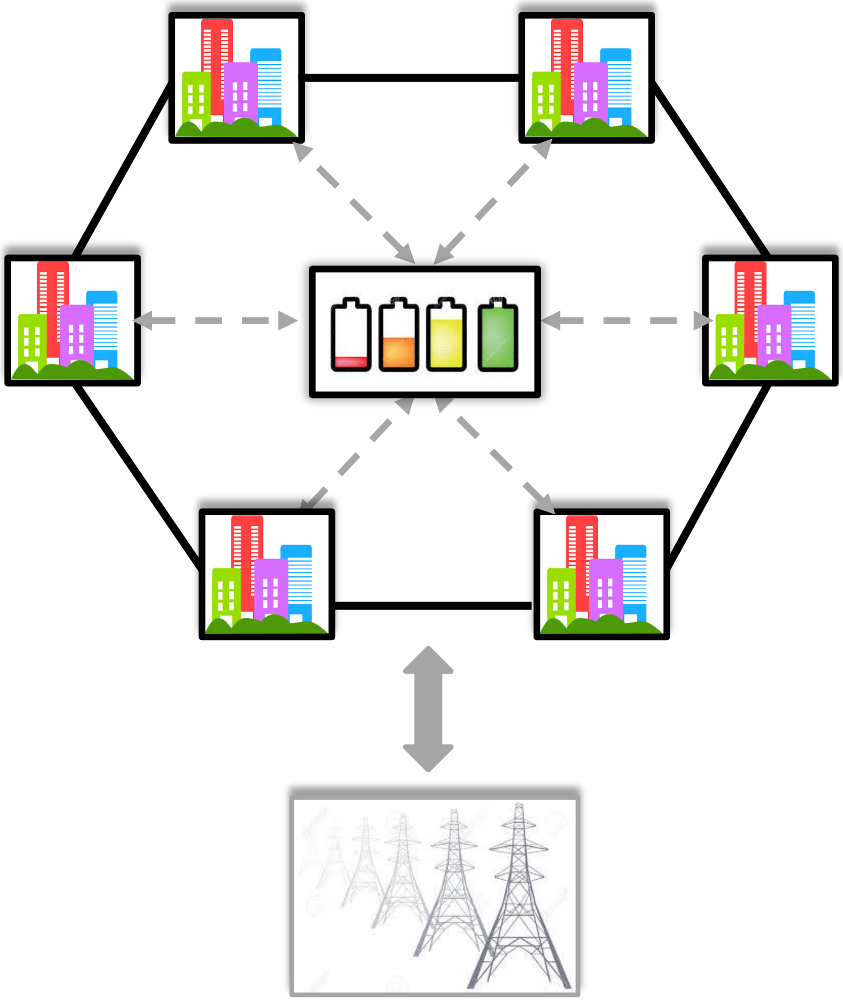


- a shared cooling network

Energy management in a building network



Building network

- connected to the main grid for electrical energy provision

Energy management in a building network

Objective:

minimize the electrical energy cost for cooling a building network by operating the chillers at their maximum efficiency, possibly shifting the cooling energy request in time by exploiting the shared cooling network.

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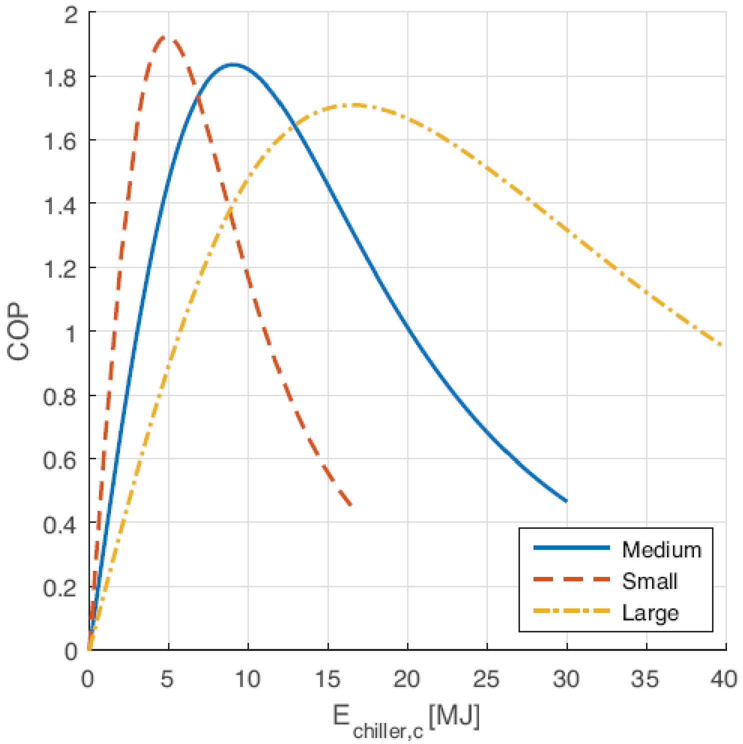
Coefficient Of Performance:

$$\mathrm{COP} = \frac{E_{ch}}{E_l}$$



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Set the energy exchanges with the cooling network of all buildings so as to

1. minimize the electrical energy cost
2. guarantee comfort conditions

over a 1-day time horizon

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Decision variables: energy exchanges with the cooling network (the storage)

Energy management in a building network

minimize **cost of all chillers electrical energy consumption**

with respect to decision variables

subject to

1. Chiller energy request = building energy request – storage energy
2. Storage dynamics
3. Storage limits, chiller limits, comfort limits

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$$\min_x \sum_i f_i(x)$$

$$s.t.\ \ x \in \bigcap_i \mathcal{X}_i$$

Constrained optimization problem

x : decision vector (decision variables along 1-day horizon)

$f_i(x)$: energy cost due to the chiller of building *i*

$\mathcal{X}_i$: comfort and actuation constraints

Optimization problem

$$\min_x \sum_i f_i(x)$$

$$s.t.\ \ x \in \bigcap_i \mathcal{X}_i$$

x : decision vector (control input along some finite horizon)

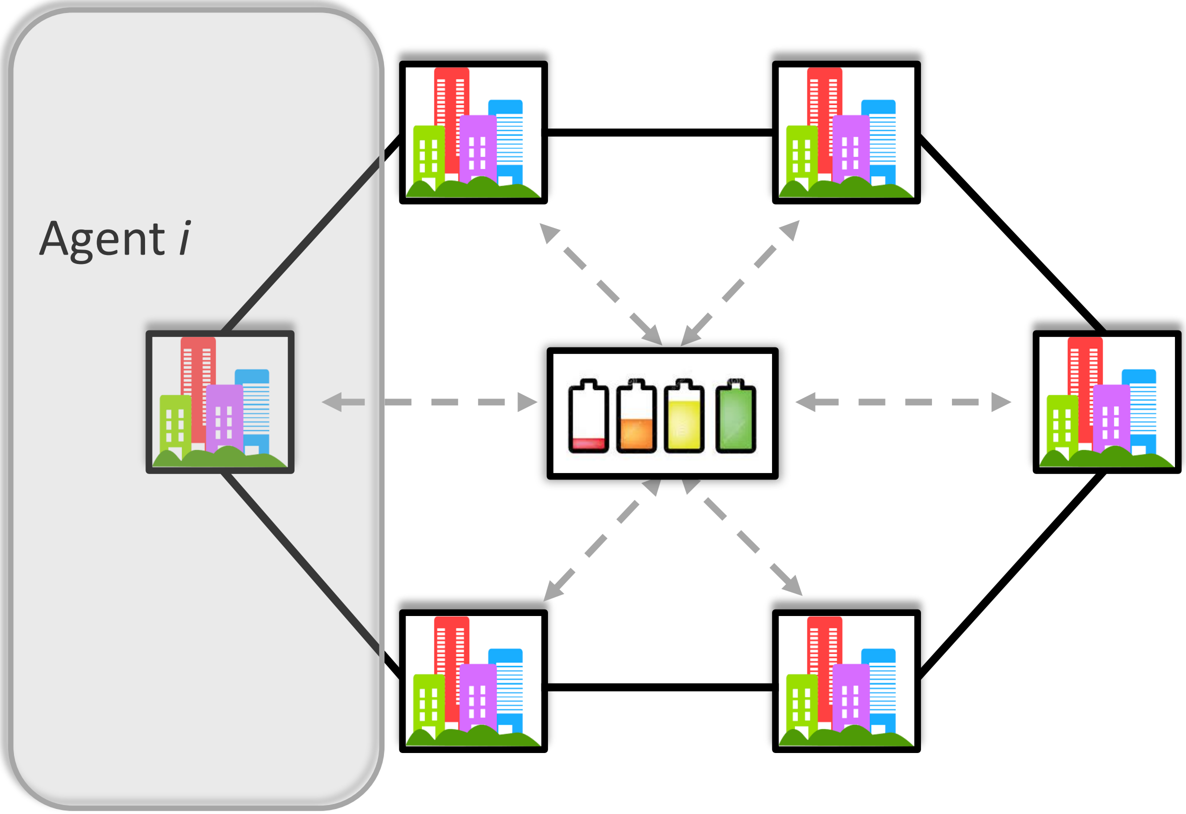
$f_i(x)$: energy cost due to the chiller of building *i*

$\mathcal{X}_i$: comfort and actuation constraints

Issues

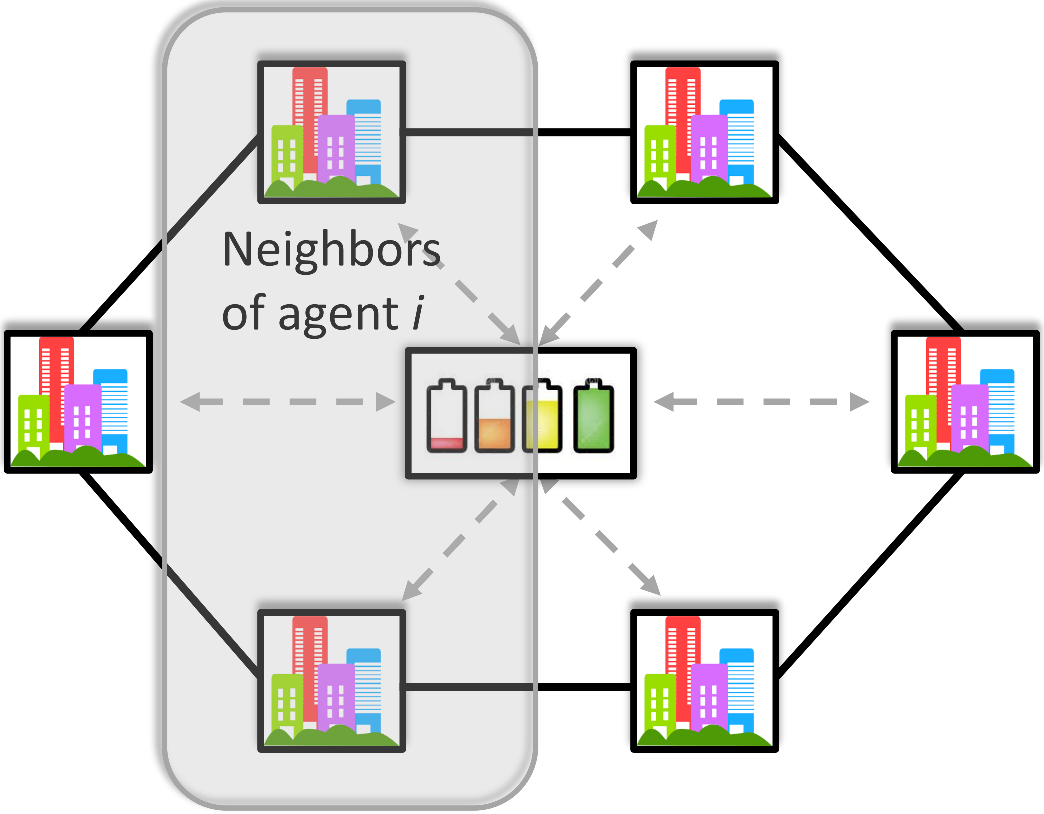
- Computation
  problem size may be too big
- Communication
  large amount of local info should be transmitted
- Information privacy
  buildings may not want to share their consumption profiles

Distributed architecture



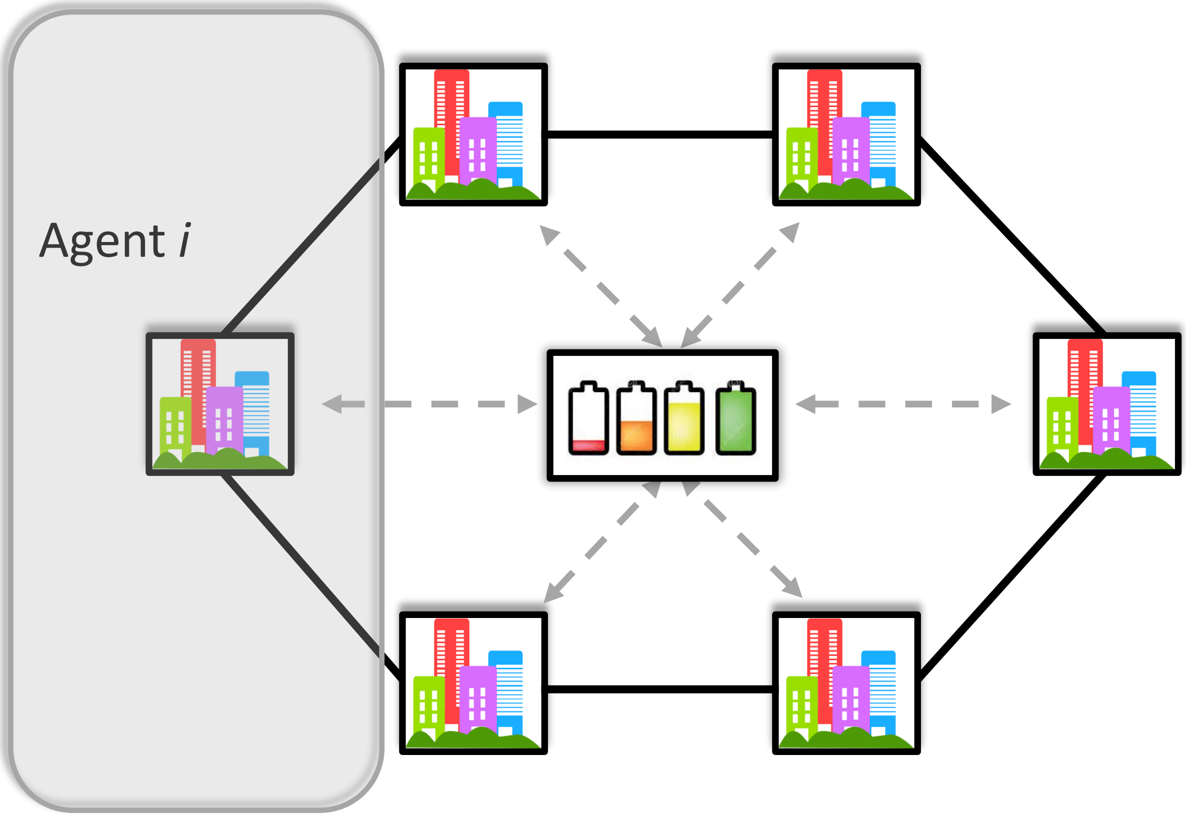
Step 1: agent *i* solves a local decision problem and makes a tentative (local) decision for *x*

Distributed architecture



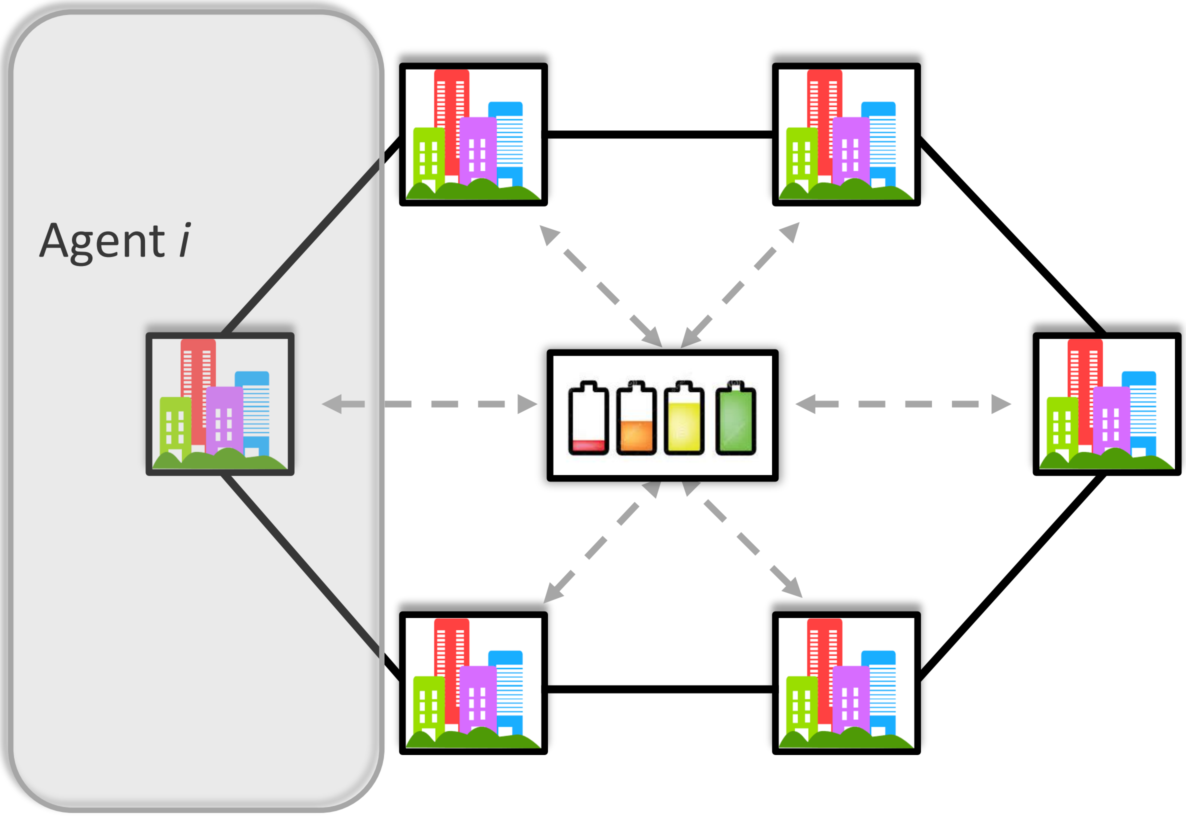
Step 2: neighbouring agents communicate their tentative decisions to agent *i*

Distributed architecture



Step 3: Agent *i* weights the received information, solves a refined problem and makes a new decision for *x*

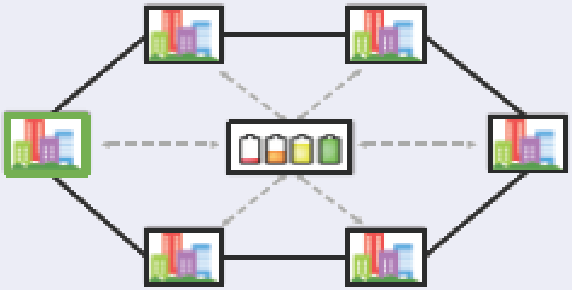
Distributed architecture



Step 3: Agent *i* weights the received information, solves a refined problem and makes a new decision for *x until convergence to some consensus solution*

Distributed optimization solution

Local problem of agent i at iteration $k+1$



$$z_i(k) = \sum_j a_j^i(k) x_j(k)$$

$$x_i(k+1) = \arg\min_{x_i \in X_i} \; f_i(x_i) + \frac{1}{2c(k)} \|x_i - z_i(k)\|^2$$

- **Information vector**

  $z_i(k) = \sum_j a_j^i(k) x_j(k)$

  $a_j^i(k)$: how agent *i* weights info of agent *j*

- **Proxy term**

  $\frac{1}{2c(k)} \|x_i - z_i(k)\|^2$: deviation from (weighted) average

  $c(k)$: trade-off between optimality and agents' disagreement

Convergence result

Theorem (consensus and optimality)

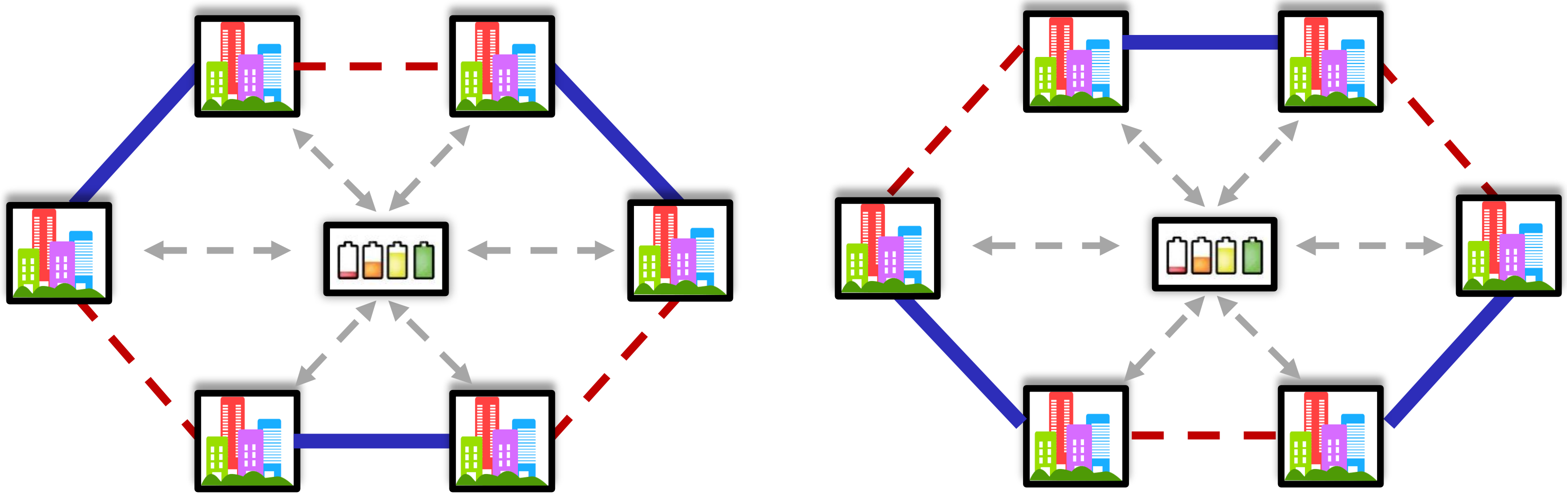
Under suitable assumptions on the weights and connectivity, if the problem is convex, then, all the agents asymptotically converge to the optimal solution of the global problem.

Distributed constrained optimization and consensus in uncertain networks via proximal minimization. IEEE TAC, vol. 63, May 2018.

Communication graph connectivity

Assumption on connectivity

- Any pair of agents communicates infinitely often, possibly through a communication graph that changes through iterations
- The intercommunication time is bounded

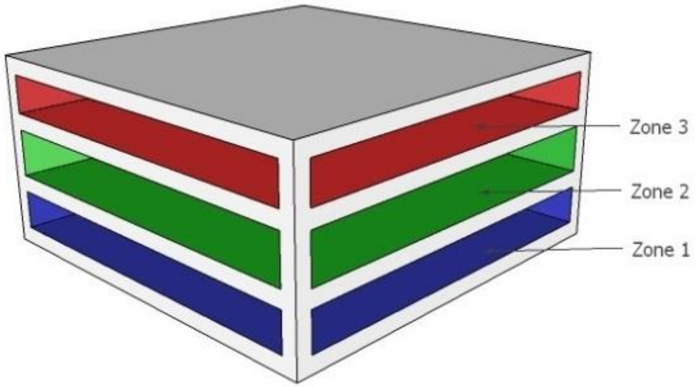


Numerical example – Building network definition

Network configuration:

3 identical buildings with three zones

and with different chillers

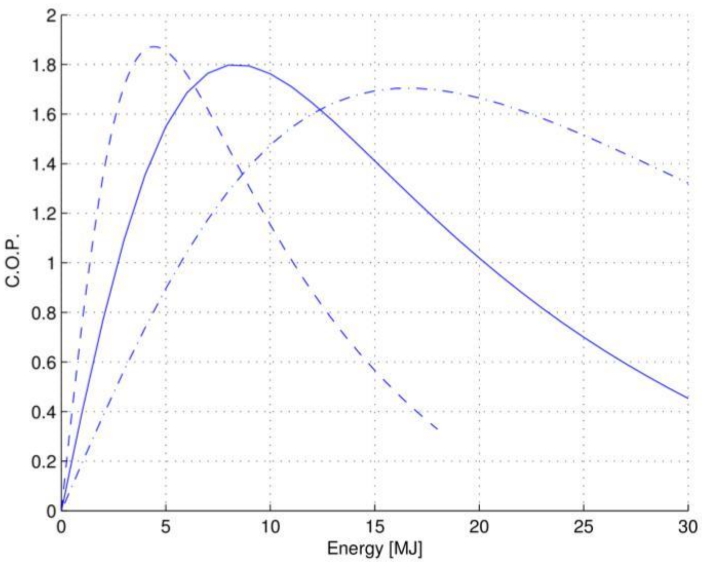


Chiller types:

'small' for building 2

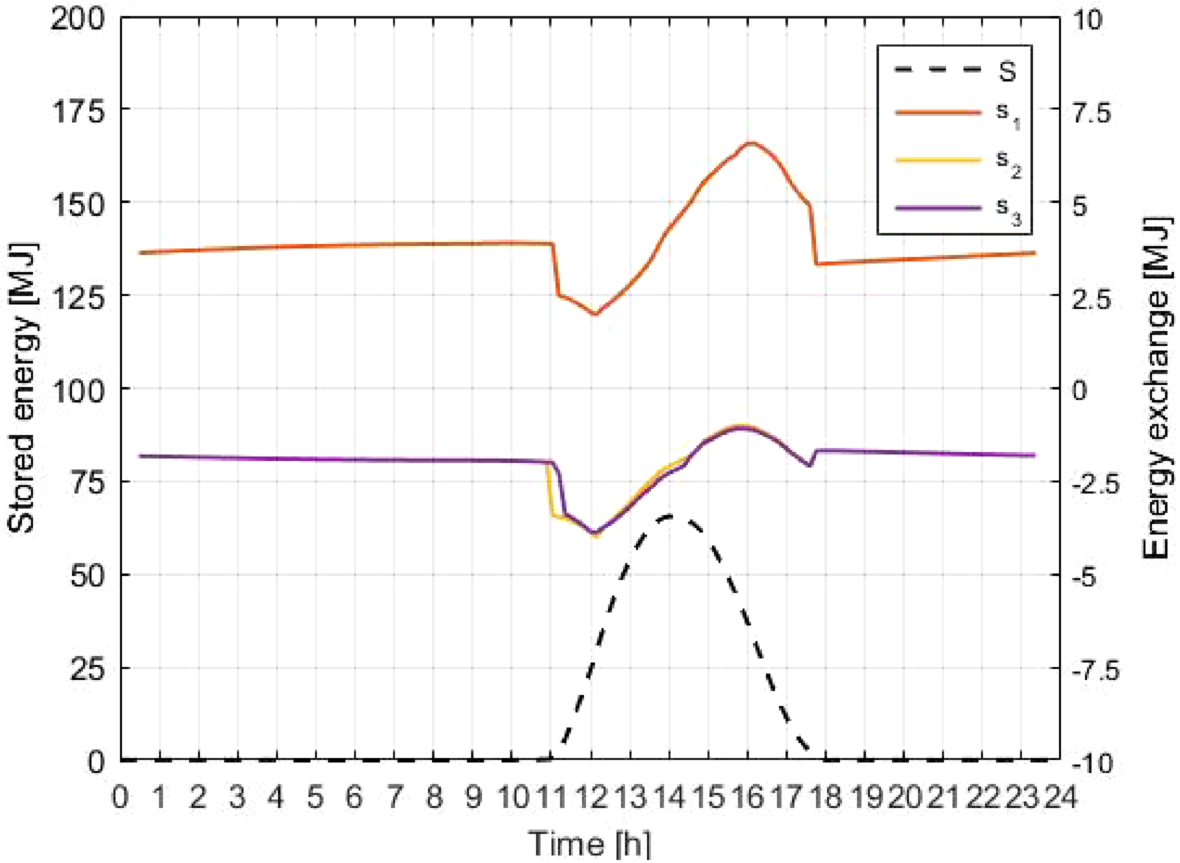
'medium' for building 1

'large' for building 3



Numerical example: convergence

solution computed by building 1



Chiller types:

‘small’ for building 2

‘medium’ for building 1

‘large’ for building 3

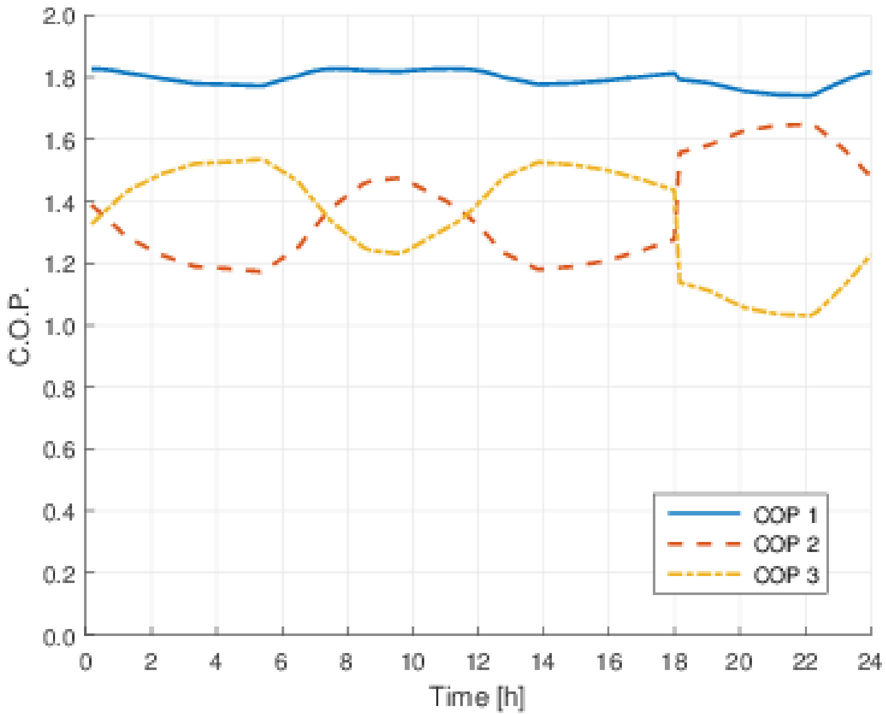
Coefficient of Performance comparison

If fixed storage, then building 2 and building 3 chillers work at suboptimal efficiency

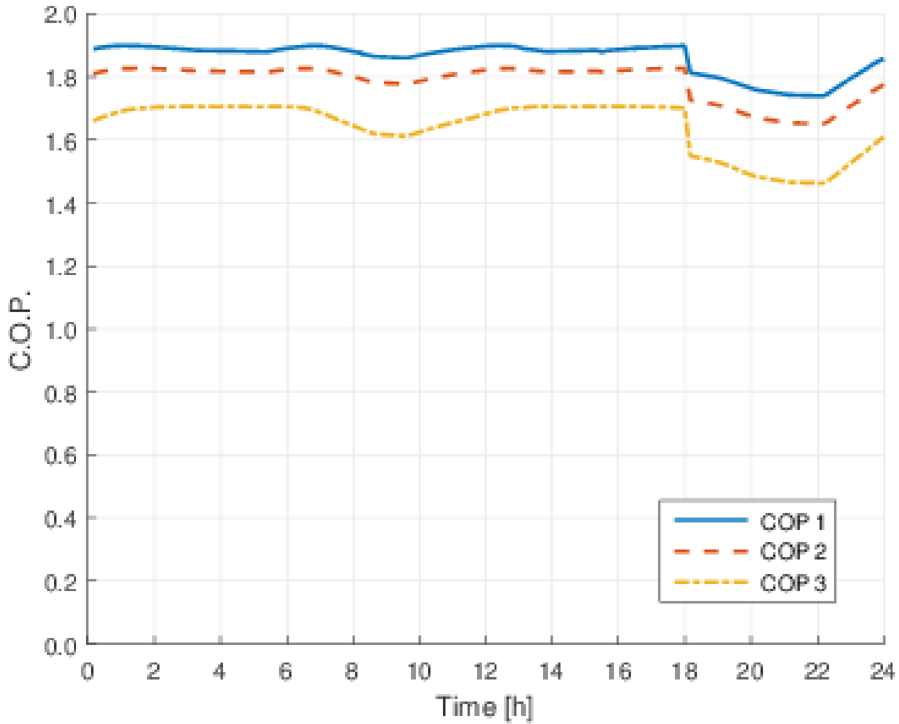
- Building 2 chiller (**small**) is overloaded
- Building 3 chiller (**large**) is underloaded

Effect of the "shared cooling system" → All chillers work close to maximum COP

COP profiles with a fixed storage



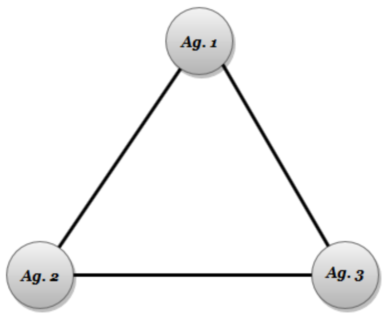
COP profiles for shared cooling network



Communication graph

Complete network graph

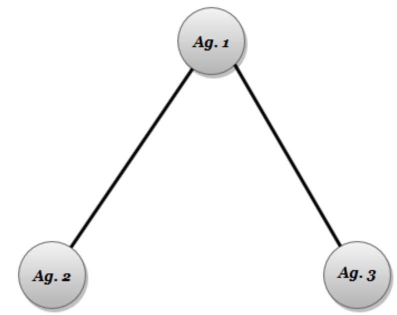
For each pair of agents (i, j) there exists a direct arc



(Strongly) Connected network graph

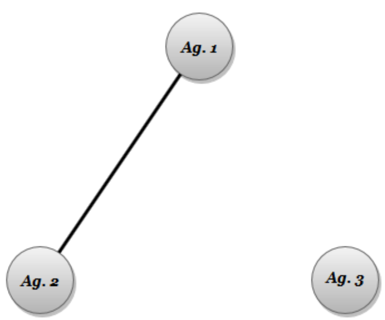
For each pair of agents (i, j)

There exists a direct path

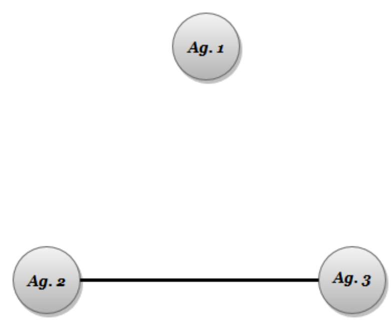


Time – varying network graph

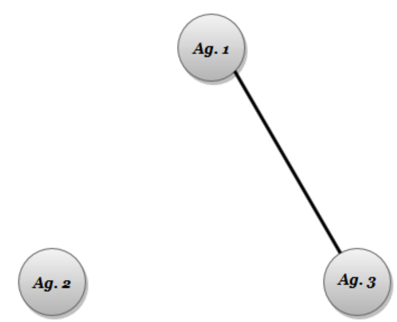
The network graph changes at each iteration k, cyclically with 3 iteration period



$k = 1$



$k = 2$



$k = 3$

Algorithm performance analysis

Communication network

- Does affect the speed of convergence and # of iterations required
  - Complete network is the fastest
  - Time – varying is the slowest
- Does **not** affect convergence properties or local problem computational time

	Complete Network	Connected Network	Time–varying Network
# of iterations	194	354	878
Average time			
Building 1	$2.743s$	$2.747s$	$2.693s$
Building 2	$2.753s$	$2.753s$	$2.687s$
Building 3	$2.738s$	$2.742s$	$2.686s$
Overall time			
Building 1	$532.160s$	$972.692s$	$2407.32s$
Building 2	$534.164s$	$968.333s$	$2399.91s$
Building 3	$531.219s$	$970.668s$	$2398.38s$

Structured optimization problems

Problem 1

$$\min_{x} \sum_{i=1}^{m} f_i(x)$$
$$s.t. \;\; x \in \bigcap_{i=1}^{m} \mathcal{X}_i$$

- local costs
- local constraints
- coupling decision vector

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Each agent knows

- local cost
- local constraint

Goal:

agree on a global minimizer

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decision coupled problem

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At each iteration k, agent i

$$z_i(k) \leftarrow \sum_j \alpha_j^i x_j(k)$$
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where

$$\tilde{f}_i(x_i) = f_i(x_i) + \frac{1}{c(k)} \|x_i - z_i(k)\|_2^2$$

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primal-based distributed algorithms

Structured optimization problems

Problem 2

$$\min_{x_1,\dots,x_m} \sum_{i=1}^{m} f_i(x_i)$$

$$s.t.\;\; x_i \in \mathcal{X}_i, i = 1, \dots, m$$
$$\sum_{i=1}^{m} g_i(x_i) \leq 0$$

- local costs
- local constraints
- local decision vectors
- coupling constraint

Structured optimization problems

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- coupling constraint

Each agent knows

- local cost
- local constraint
- local coupling contribution

Goal:

minimize global cost

Structured optimization problems

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Goal:

minimize global cost

- local costs
- local constraints
- local decision vectors
- coupling constraint

constraint coupled problem

Structured optimization problems

Introduce the Lagrangian function

$$L(x,\lambda) = \sum_{i=1}^{m} L_i(x_i,\lambda) = \sum_{i=1}^{m} \{f_i(x_i) + \lambda^\top g_i(x_i)\}$$

where λ is a vector of Lagrange multipliers and define the dual function

$$\varphi(\lambda) = \min_{x\in X} L(x,\lambda) = \sum_{i=1}^{m} \varphi_i(\lambda) = \sum_{i=1}^{m} \min_{x_i\in X_i} L_i(x_i,\lambda)$$

Then, if the problem is convex, the dual problem

$$\max_{\lambda\geq 0} \sum_{i=1}^{m} \varphi_i(\lambda)$$

provides the optimal value for the primal problem.

Lectures on Math tools: duality theory

Structured optimization problems

Problem 2

$$\min_{x_1,\dots,x_m} \sum_{i=1}^{m} f_i(x_i)$$

$$s.t.\;\; x_i \in \mathcal{X}_i, i = 1, \dots, m$$
$$\sum_{i=1}^{m} g_i(x_i) \leq 0$$

- local costs
- local constraints
- local decision vectors
- coupling constraint

At each iteration k, agent i

$$\ell_i(k) \leftarrow \sum_j \alpha_j^i \lambda_j(k)$$

$$\lambda_i(k+1) \leftarrow \arg\max_{\lambda_i \geq 0} \; \varphi_i(\lambda_i)$$

where

$$\varphi_i(\lambda_i) = \lambda_i^\top g_i(x_i(k+1)) - \frac{1}{c(k)} \|\lambda_i - \ell_i(k)\|_2^2$$

Structured optimization problems

Problem 2

$$\min_{x_1, \ldots, x_m} \sum_{i=1}^{m} f_i(x_i)$$

$$s.t. \;\; x_i \in \mathcal{X}_i, i = 1, \ldots, m$$
$$\sum_{i=1}^{m} g_i(x_i) \leq 0$$

- local costs
- local constraints
- local decision vectors
- coupling constraint

At each iteration k, agent i

$$\ell_i(k) \leftarrow \sum_j \alpha_j^i \lambda_j(k)$$

$$x_i(k+1) \leftarrow \arg\min_{x_i \in \mathcal{X}_i} \tilde{f}_i(x_i)$$

$$\lambda_i(k+1) \leftarrow \arg\max_{\lambda_i \geq 0} \varphi_i(\lambda_i)$$

where

$$\tilde{f}_i(x_i) = f_i(x_i) + \ell_i(k)^\top g_i(x_i)$$

$$\varphi_i(\lambda_i) = \lambda_i^\top g_i(x_i(k+1)) - \frac{1}{c(k)} \|\lambda_i - \ell_i(k)\|_2^2$$

duality-based distributed algorithms

Theoretical results

Theorem (consensus and optimality)

Under suitable assumptions, all the agents asymptotically converge to the optimal solution of the dual of global problem 2. Moreover, they are able to recover the optimal primal solution.

Dual decomposition for multi-agent distributed optimization with coupling constraints. Automatica, vol. 84, pp. 149-158, October 2017.

Structured optimization problems

Problem 1

$$\min_{x} \sum_{i=1}^{m} f_i(x)$$
$$s.t. \;\; x \in \bigcap_{i=1}^{m} \mathcal{X}_i$$

decision coupled

Problem 2

$$\min_{x_1,\, \dots,\, x_m} \sum_{i=1}^{m} f_i(x_i)$$
$$s.t. \;\; x_i \in \mathcal{X}_i, i = 1, \dots, m$$
$$\sum_{i=1}^{m} g_i(x_i) \leq 0$$

constraint coupled

Structured optimization problems

Problem 1

$$\min_{x} \sum_{i=1}^{m} f_i(x)$$
$$s.t.\ \ x \in \bigcap_{i=1}^{m} \mathcal{X}_i$$

decision coupled

primal-based algorithm

Problem 2

$$\min_{x_1, \dots, x_m} \sum_{i=1}^{m} f_i(x_i)$$
$$s.t.\ \ x_i \in \mathcal{X}_i, i = 1, \dots, m$$
$$\sum_{i=1}^{m} g_i(x_i) \le 0$$

constraint coupled

duality-based algorithm

Math tools from convex analysis, optimization and duality theory to study convergence and optimality, extension to the uncertain case through a data-driven approach.

Structured optimization problems

Problem 3

$$\min_{x_1,\ldots,x_m} \sum_{i=1}^{m} f_i(x_1,\ldots,x_m)$$
$$s.t.\ \ x_i \in \mathcal{X}_i, i = 1,\ldots,m$$

- local constraints
- local decision vectors
- coupled local costs

Problem 3

$$\min_{x_1,\,\dots,\,x_m} \sum_{i=1}^{m} f_i(x_1,\dots,x_m)$$
$$s.t.\ \ x_i \in \mathcal{X}_i, i = 1,\dots,m$$

- local constraints
- local decision vectors
- coupled local costs

Each agent knows

- local constraint
- local cost

Goal:

minimize global cost

Structured optimization problems

Problem 3

$$\min_{x_1,\dots,x_m} \sum_{i=1}^{m} f_i(x_1,\dots,x_m)$$
$$s.t.\ \ x_i \in \mathcal{X}_i, i = 1,\dots,m$$

- local constraints
- local decision vectors
- coupled local costs

Each agent knows

- local constraint
- local cost

Goal:

minimize global cost

cost coupled problem

Electric vehicle charging

Design a charging strategy for a fleet of m electric vehicles over a finite horizon

Electric vehicle charging

Design a charging strategy for a fleet of m electric vehicles over a finite horizon

Electric vehicle charging

Design a charging strategy for a fleet of m electric vehicles over a finite horizon

- A central authority broadcast an energy price incentive based on the total energy consumption
- Agents (EVs) optimize their charging strategy based on the energy price incentive

EV charging – Cooperative set-up

Agents cooperate to achieve a **social welfare** optimizing solution

EV charging – Cooperative set-up

Agents cooperate to achieve a **social welfare** optimizing solution

Objective

- design a charging strategy over a finite horizon $t = 0, \dots, T-1$, so as to minimize the cost for the whole fleet

Decision variables

- x^{it} represents the charging rate of agent i at time t

EV constraints

- $\underline{x}^{it}$ and $\overline{x}^{it}$ lower and upper bounds on x^{it}
- γ^i is the charging level to be reached by agent i at the end of the time horizon

Energy cost

- $\sum_{i=1}^{m} x^{it}$ total demand at time t
- $p_t(\sum_{i=1}^{m} x^{it})$ price of energy depends on the total demand

EV charging – Cooperative set-up

EV charging problem can be formulated as a constrained optimization:

$$\min_{\{x^1,\ldots,x^m\}} \sum_{i=1}^{m} f_i(x^1,\ldots,x^m) = \sum_{i=1}^{m} \sum_{t=0}^{T-1} p_t\left(\sum_{i=1}^{m} x^{it}\right) x^{it}$$

$$s.t. \quad \underline{x}^{it} \le x^{it} \le \overline{x}^{it} \quad \forall t = 0,\ldots,T-1 \;\; \forall i = 1,\ldots,m$$

$$\sum_{t=0}^{T-1} x^{it} = \gamma^i \qquad \forall i = 1,\ldots,m$$

- $x^i = \left(x^{i0}, x^{i1}, \ldots, x^{iT-1}\right)$ is the decision vector of agent i
- the objective function represents the total cost given by the sum of the costs $f_i(x^1,\ldots,x^m)$ over all the agents
- price of energy per time slot depends on the total demand: $p_t\left(\sum_{i=1}^{m} x^{it}\right)$
- constraints are decoupled and ensure that each EV is fully charged, and that power limitations are respected

EV charging – Cooperative set-up

$$\min_{x^1,\ldots,x^m} \sum_{i=1}^{m} f_i(x^1,\ldots,x^m)$$
$$s.t.\ \ x^i \in \mathcal{X}^i, i = 1,\ldots,m$$

EV charging – Cooperative set-up

$$\min_{x^1,\dots,x^m} \sum_{i=1}^{m} f_i(x^1, \dots, x^m)$$
$$s.t.\ \ x^i \in \mathcal{X}^i, i = 1, \dots, m$$

- electric vehicles need to charge their battery and decide how much to charge along some time horizon
- price depends on the overall demand and is set by some authority

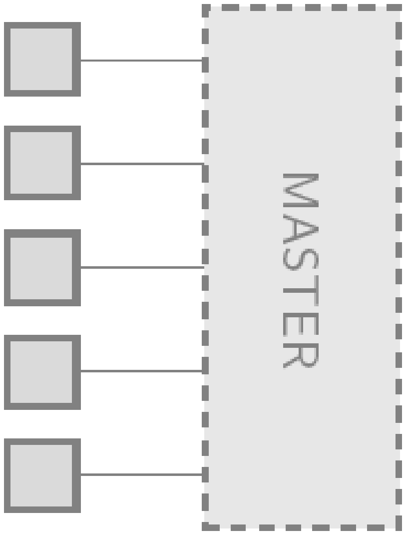
EV charging – Cooperative set-up

$$\min_{x^1,\ldots,x^m} \sum_{i=1}^{m} f_i(x^1, \ldots, x^m)$$
$$s.t.\ \ x^i \in \mathcal{X}^i, i = 1, \ldots, m$$

- electric vehicles need to charge their battery and decide how much to charge along some time horizon
- price depends on the overall demand and is set by some authority

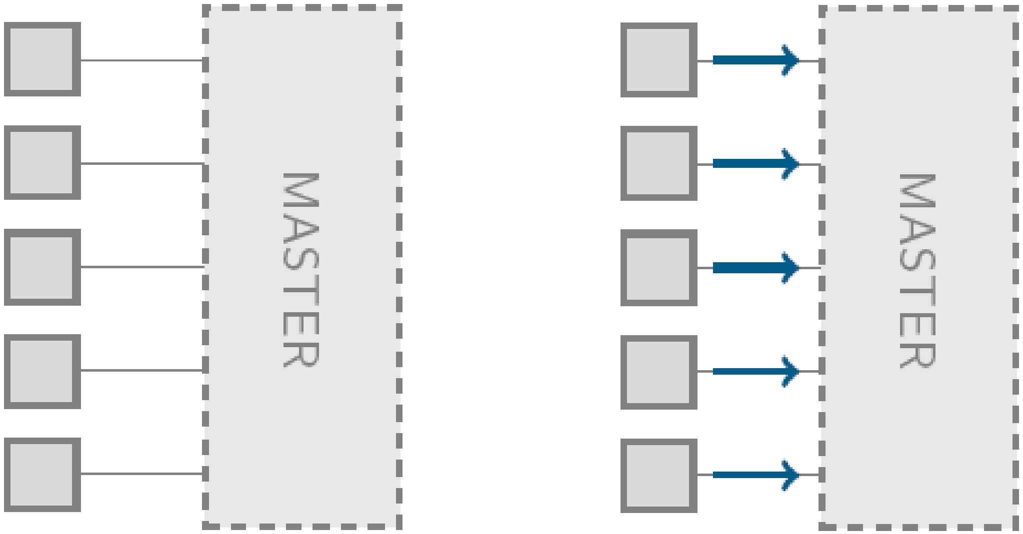
decision making problem naturally addressed according to a decentralized optimization scheme where the authority is the central unit collecting info from all agents broadcasting the new price

Decentralized, parallel, master/slave scheme



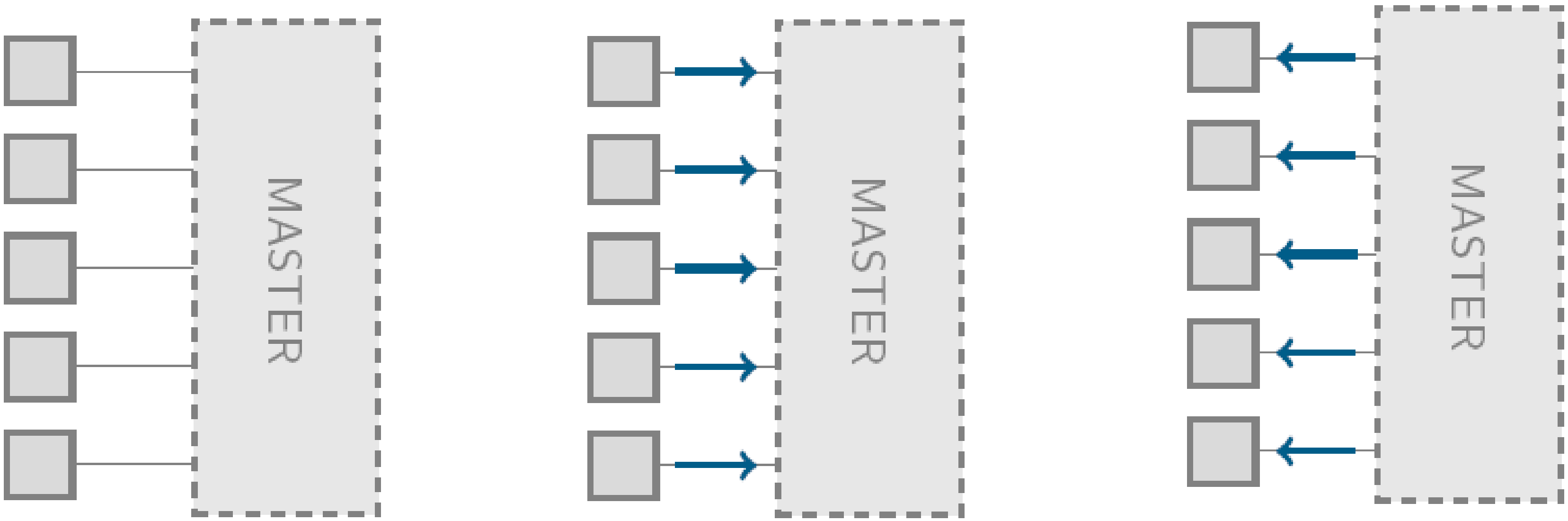
- Agents make local computations

Decentralized, parallel, master/slave scheme



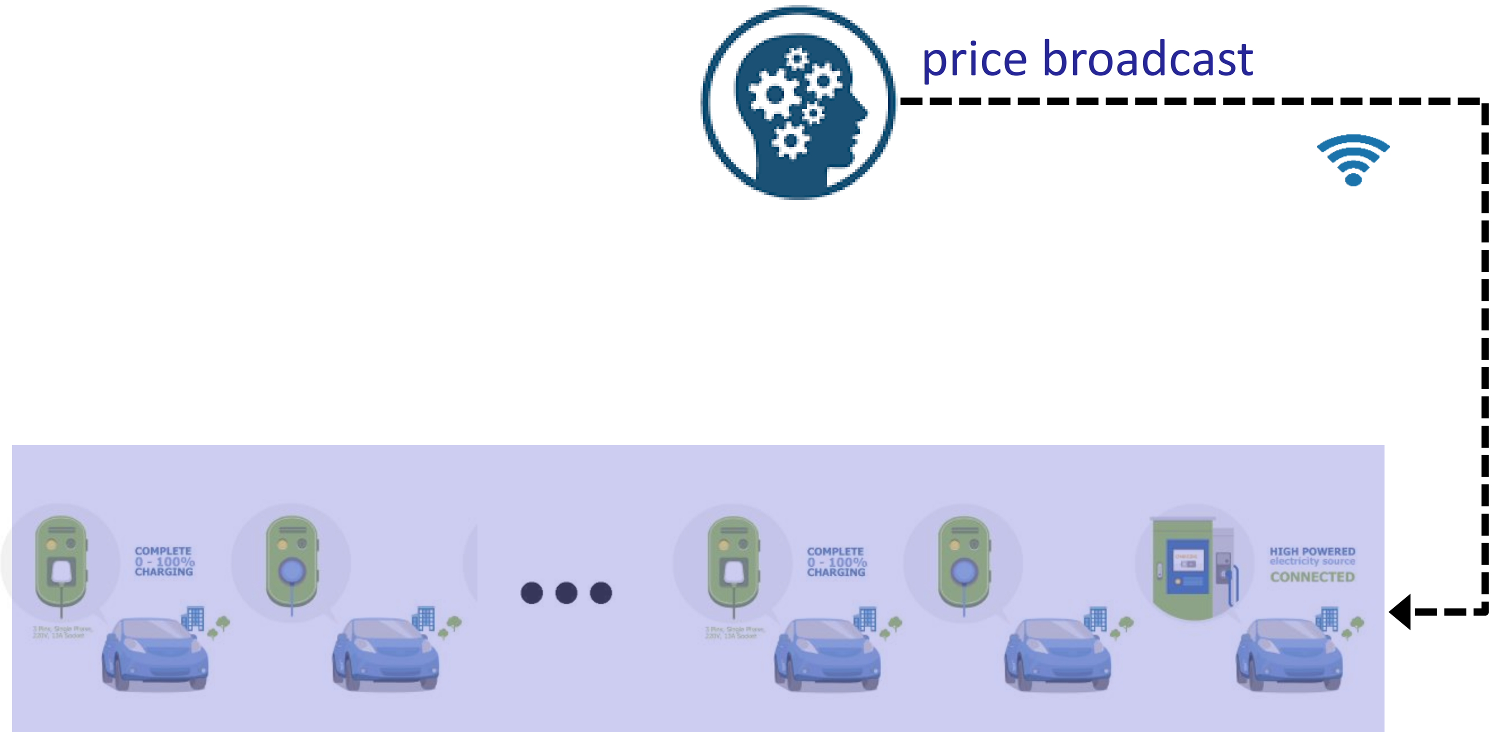
- Agents make local computations
- They communicate the outcome to the central unit (master)

Decentralized, parallel, master/slave scheme



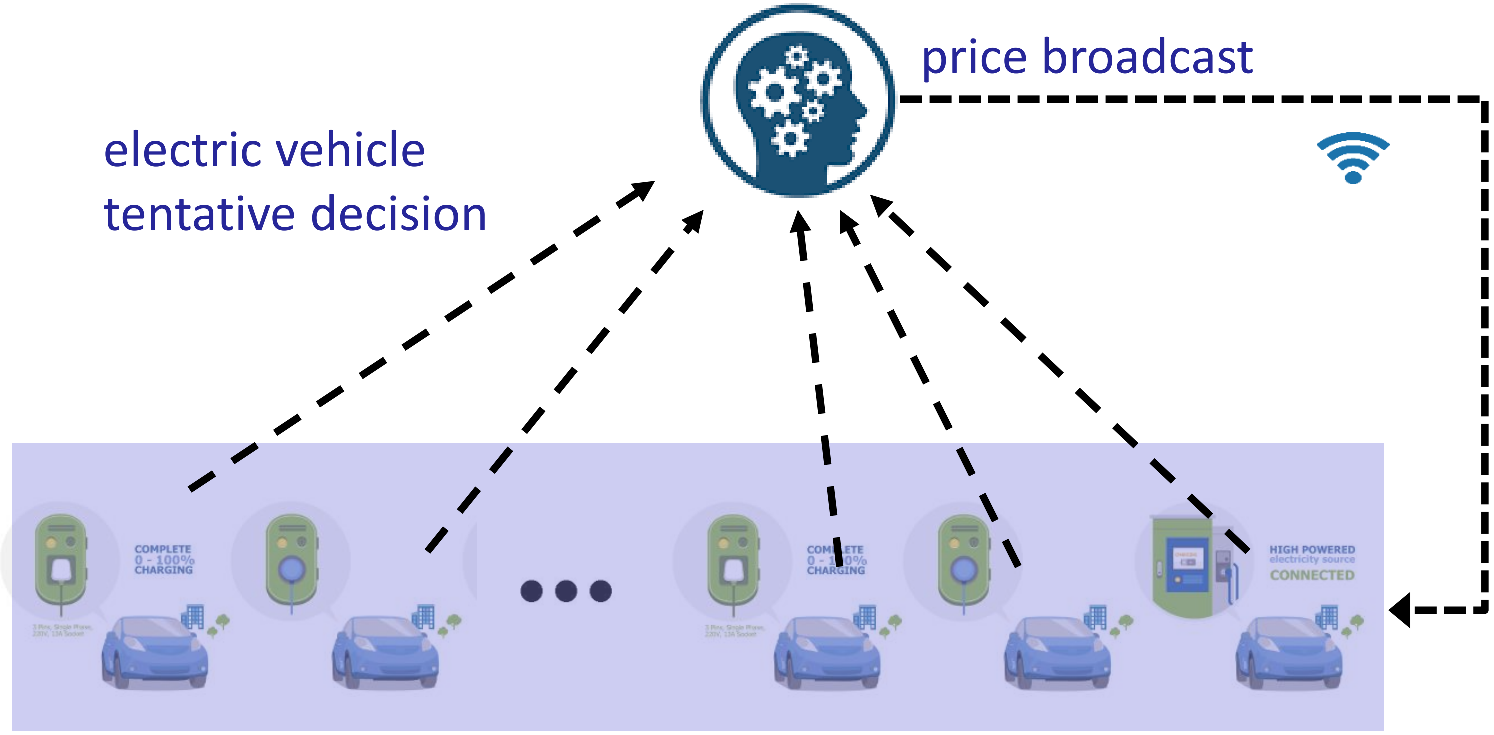
- Agents make local computations
- They communicate the outcome to the central unit (master)
- The master provide back some updated info based on the ones received from the agents

EV charging – Cooperative set-up



Aggregator/central authority sends a price incentive to all agents, which depends on their consumption levels $x^1, \dots, x^m$

EV charging – Cooperative set-up



Agents solve some local problem and send to the aggregator/central authority an update of their consumption levels $x^1, \dots, x^m$

A new price is calculated and the process is repeated

EV charging – Cooperative set-up

$$\boldsymbol{x}(k) = \begin{bmatrix} x^1(k) \\ \vdots \\ x^N(k) \end{bmatrix} \quad \Longrightarrow \quad \boldsymbol{x}(k+1) \in \mathcal{T}(\boldsymbol{x}(k))$$

EV charging – Cooperative set-up

$$x(k) = \begin{bmatrix} x^1(k) \\ \vdots \\ x^N(k) \end{bmatrix} \quad \Longrightarrow \quad x(k+1) \in \mathcal{T}(x(k))$$

If limit point of the resulting iterative method can be characterized as a fixed point of an operator, then, decentralized optimization rewrites as a fixed point algorithm

Lecture on Decomposition optimization algorithms

POLITECNICO DI MILANO

Distributed algorithms for optimization and control over networks

DEIB PhD Course, Politecnico di Milano

February 10-14, 2020